Gr 9 Maths: Content Area 2
Patterns, Algebra & Graphs
ANSWERS

- Patterns
- Algebraic Expressions
- Factorisation
- Algebraic Equations
- Graphs

PATTERNS

1.1 **C** ➤ . . . 1^2 ; 3^2 ; 5^2 ; $\mathbf{7^2}$

1.2 **C** ➤ . . . $18 \xrightarrow{+18} 36 \xrightarrow{+18} \mathbf{54} \xrightarrow{+18} 72 \xrightarrow{+18} \mathbf{90}$

1.3 **A** ➤ . . . $\frac{1}{2^0}$; $\frac{1}{2^1}$; $\frac{1}{2^2}$; $\mathbf{\frac{1}{2^3}}$; $\frac{1}{2^4}$

1.4 **A** ➤ . . . $\frac{1}{3}$; $\mathbf{\frac{1}{6}}$; $\frac{1}{12}$; $\frac{1}{24}$; $\frac{1}{48}$

1.5 **D** ➤ . . . 1^2+2 ; 2^2+2 ; 3^2+2 ; 4^2+2 ; $\mathbf{5^2+2}$

2.1 a) $T_n = 3n$:

3 ; 6 ; 9 ➤ . . . $3(\mathbf{1})$; $3(\mathbf{2})$; $3(\mathbf{3})$

b) $T_n = 5n$:

5 ; 10 ; 15 ➤ . . . $5(\mathbf{1})$; $5(\mathbf{2})$; $5(\mathbf{3})$

c) $T_n = 3n + 1$:

4 ; 7 ; 10 ➤ . . . $3(\mathbf{1}) + 1$; $3(\mathbf{2}) + 1$; $3(\mathbf{3}) + 1$

d) $T_n = 5n - 2$:

3 ; 8 ; 13 ➤ . . . $5(\mathbf{1}) - 2$; $5(\mathbf{2}) - 2$; $5(\mathbf{3}) - 2$

e) $T_n = n^2$:

1 ; 4 ; 9 ➤ . . . $(\mathbf{1})^2$; $(\mathbf{2})^2$; $(\mathbf{3})^2$

f) $T_n = n^3$:

1 ; 8 ; 27 ➤ . . . $(\mathbf{1})^3$; $(\mathbf{2})^3$; $(\mathbf{3})^3$

2.2 a) $T_{12} = 3(\mathbf{12}) = \mathbf{36}$ ➤

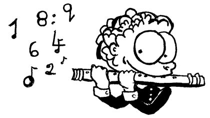
b) $T_{12} = 5(\mathbf{12}) = \mathbf{60}$ ➤

c) $T_{12} = 3(\mathbf{12}) + 1 = \mathbf{37}$ ➤

d) $T_{12} = 5(\mathbf{12}) - 2 = \mathbf{58}$ ➤

e) $T_{12} = (\mathbf{12})^2 = \mathbf{144}$ ➤

f) $T_{12} = (\mathbf{12})^3 = \mathbf{1\ 728}$ ➤



3.1 **y = 3*x*** ➤

3.2 **a = 7** ➤ . . . $\mathbf{7} \times 3 = 21$

b = 30 ➤ . . . $10 \times 3 = \mathbf{30}$

4.1 The constant difference = **7** ➤

4.2 3 ; 10 ; 17 ; 24 ; 31 ; **38** ; **45** ➤

4.3 The constant **difference** is **7** . . . *see Question 4.1*

So, write down **the multiples of 7** . . . where $T_n = 7n$:

7 ; 14 ; 21 ; 28 ; 35 ; . . .

and compare the given sequence :

3 ; 10 ; 17 ; 24 ; 31 ; . . .

Each term is **4 less** than the multiples of 7.

$\therefore$ $\mathbf{T_n = 7n - 4}$ ➤

5.1

Position in pattern	1	2	3	4	5
Term	1	8	27	**64**	**125**

➤

5.2 $\mathbf{T_n = n^3}$ ➤

5.3 If $T_n = 512$, then **n = 8** ➤

. . . $512 = 2^9$

$= 2^3 \times 2^3 \times 2^3$

$= 8 \times 8 \times 8$

$= \mathbf{8^3}$ ➤

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

6.1 7 ; 11 ; 15 ; **19** ; **23** ➤

6.2 The common **difference** is **4**

So, compare the **multiples of 4** . . . where $T_n = 4n$:

4 ; 8 ; 12 ; 16 ; . . .

to the given sequence :

7 ; 11 ; 15 ; 19 ; . . .

Each term is **3 more** than the multiples of 4.

$\therefore$ $\mathbf{T_n = 4n + 3}$ ➤ . . . $7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$

6.3 $T_{50} = 4(50) + 3$

$= \mathbf{203}$ ➤

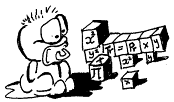
7.1 3 ; 8 ; 13 ; **18** ; **23**

7.2 Each term is 5 more than the previous term

7.3 Compare $T_n = 5n$: 5 ; 10 ; 15 ; . . .
to : 3 ; 8 ; 13 ; . . .

Each term is **2 less** than the multiples of 5

$\therefore$ **T_n = 5n – 2**



7.4 38 is 2 less than 40 ; 40 = 5 **x 8**

$\therefore$ 38 is the **8th** term

OR : Solve the equation :

$5n - 2 = 38$ . . . *the n^{th} term = 38*

Add 2: $\therefore\ 5n = 40$

Divide by 5: $n = 8$

$\therefore$ The **8th** term

8.1

Figure	1	2	3	4
Number of sides	5	9	**13**	**17**

8.2 Each figure has four more sides than the previous figure

8.3 **T_n = 4n + 1** . . . $5 \overset{+4}{;} 9 \overset{+4}{;} 13 \overset{+4}{;} 17$

*& each term is **1 more** than the multiples of 4*

9.

Figure	1	2	3
Number of matchsticks	6	9	12

9.1 Number of matchsticks in Figure 4 = **15**

9.2 **T_n = 3n + 3** . . . $6 \overset{+3}{;} 9 \overset{+3}{;} 12$

*& each term is **3 more** than the multiples of 3*

9.3 T_{20} = 3(**20**) + 3
= **63 matchsticks**

10.1

Figure	1	2	3	4
Number of black tiles	1	2	3	4
Number of white tiles	6	**10**	**14**	**18**

10.2 **T_n = 4n + 2** . . . $6 \overset{+4}{;} 10 \overset{+4}{;} 14 \overset{+4}{;} 18$

*& each term is **2 more** than the multiples of 4*

---

11. Look at the pattern formed by the first numbers of each line :

1 ; 4 ; 9 ; . . . the squares !

row **1** row **2** row **3**
(**1**^{2}) (**2**^{2}) (**3**^{2})

$\therefore$ The first number in the **20th** row will be **20**^{2} = **400**

NOTES

ALGEBRAIC EXPRESSIONS

Terminology

1.1 $\mathbf{-8}$ ◄ . . . $\mathbf{-8}x^2$

1.2 $\mathbf{-7}$ ◄ . . . *the term with no variable*

1.3 $\mathbf{-8x^2 + 2x - 7}$ ◄

1.4 $\mathbf{1}$ ◄ . . . $2x^{\mathbf{1}}$

1.5 $2x - 7 - 8x^2$

$$x = \frac{1}{2}: \quad 2\left(\frac{1}{2}\right) - 7 - 8\left(\frac{1}{2}\right)^2$$
$$= \left(\frac{2}{1}\right)\left(\frac{1}{2}\right) - 7 - \left(\frac{8}{1}\right)\left(\frac{1}{4}\right)$$
$$= 1 - 7 - 2$$
$$= \mathbf{-8} ◄$$

---

2. **B** ◄ . . . *x is part of the fraction $\frac{x - y}{3}$, which can also be written as $\frac{1}{3}(x - y) = \frac{1}{3}x - \frac{1}{3}y$.*

$\therefore$ The coefficient of x is $\frac{1}{3}$.

[The two variables are x and y.]

Substitution

3.1 $2x^3 - 3x^2 + 9x + 2$

$$\underline{x = -2}: \quad 2(-2)^3 - 3(-2)^2 + 9(-2) + 2$$
$$= 2(-8) - 3(4) + 9(-2) + 2$$
$$= -16 - 12 - 18 + 2$$
$$= \mathbf{-44} ◄$$

3.2 $y = 2x^2 - 3x + 5$

$$\underline{x = -1}: \quad y = 2(-1)^2 - 3(-1) + 5$$
$$= 2 + 3 + 5$$
$$= \mathbf{10} ◄$$

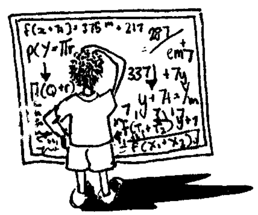
3.3 $\frac{5ac}{b}$

$$= \frac{\frac{5}{1}\left(\frac{2}{1}\right)\left(\frac{1}{2}\right)}{(-3)}$$ . . . *it is useful to put whole numbers over 1 when fractions are involved*

$$= \frac{5(1)}{-3}$$
$$= -\frac{\mathbf{5}}{\mathbf{3}} ◄$$

3.4 $3x^2 - 2xy - y^2$

$$= 3(2)^2 - 2(2)(-3) - (-3)^2$$
$$= 3(4) - 2(-6) - (9)$$
$$= 12 + 12 - 9$$
$$= \mathbf{15} ◄$$



Addition, Subtraction, Multiplication and Division

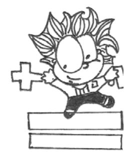
4.1 $(2b - 3a - c) + (a - 4b + 2c)$

$$= 2b - 3a - c + a - 4b + 2c$$
$$= -3a + a + 2b - 4b - c + 2c$$
$$= \mathbf{-2a - 2b + c} ◄$$

OR: $\quad -3a + 2b - c$
Add $\quad \underline{\ a - 4b + 2c}$
$\quad -2a - 2b + c$ ◄

4.2 $-4x^2(5x^2 - 3x)$

$$= \mathbf{-20x^4 + 12x^3} ◄$$



Distributive Property:
$a(b + c) = ab + ac$

4.3 $\frac{8a + 16a^2 - 4a^3}{2a}$

$$= \frac{8a}{2a} + \frac{16a^2}{2a} - \frac{4a^3}{2a}$$ . . . *Each **term** in the numerator must be divided by 2a.*

$$= \mathbf{4 + 8a - 2a^2} ◄$$

4.4 $-3(x)(x) + 2x(-x)$

$$= -3x^2 - 2x^2$$ . . . *LIKE TERMS* ☺
$$= \mathbf{-5x^2} ◄$$

4.5 $-3m^2n(4m - 3mn^5 + 2n)$ . . . *Distributive Property*

$$= \mathbf{-12m^3n + 9m^3n^6 - 6m^2n^2} ◄$$

4.6 $3ab - (-2ab)$

$$= 3ab + 2ab$$ . . . *LIKE TERMS* ☺
$$= \mathbf{5ab} ◄$$

5.1 $(3x)^3 + 2x^3$ . . . $(3x)^3 = 3^3 . x^3$

$= 27x^3 + 2x^3$ . . . *LIKE TERMS* ☺

$= \mathbf{29x^3}$ ◄

5.2 $(2x)^2 \times 3x^2$ . . . $(2x)^2 = 2x \times 2x$

$= 4x^2 \times 3x^2$

$= \mathbf{12x^4}$ ◄

5.3 $(a^2b^3)^2 . ab^2$

$= a^4b^6 . ab^2$

$= \mathbf{a^5b^8}$ ◄

5.4 $2^5 - 1^5$ . . . $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$= 32 - 1$ . . . $1^5 = 1 \times 1 \times 1 \times 1 \times 1$

$= \mathbf{31}$ ◄

Fractions (+, −, ×, ÷)

5.5 $\frac{x \times 5}{2 \times 5} + \frac{x \times 2}{5 \times 2}$

$= \frac{5x + 2x}{10}$

$= \mathbf{\frac{7x}{10}}$ ◄

When we add or subtract fractions, we must determine a common denominator

5.6 $\frac{5a \times 3}{8 \times 3} - \frac{5a \times 2}{12 \times 2}$

$= \frac{15a - 10a}{24}$

$= \mathbf{\frac{5a}{24}}$ ◄

NB: *'Keep' the denominator! Do not multiply by it.*

5.7 $\frac{a^2b^2}{ac^2} \times \frac{4a^2bc}{20b^3}$

When we multiply fractions, we may cancel FACTORS.

$= \frac{ab^2}{c^2} \times \frac{a^2c}{5b^2}$

$= \mathbf{\frac{a^3}{5c}}$

NB: Understand the expression

$\frac{a \times a \times b \times b}{a \times c \times c} \times \frac{4 \times a \times a \times b \times c}{20 \times b \times b \times b}$

. . . and then cancel!

5.8 $\frac{6x^5}{x^4} - \frac{15x^3}{3x^2}$

$= 6x - 5x$ . . . *LIKE TERMS* ☺

$= \boldsymbol{x}$ ◄

NB: Understand the expression

$\frac{6 \times x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}} - \frac{{}^{5}\cancel{15} \times x \times \cancel{x} \times \cancel{x}}{\cancel{3} \times \cancel{x} \times \cancel{x}}$

5.9 $\frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4}$

$= \frac{2x + 1 - 2(x+2) - 1}{4}$

$= \frac{2x + 1 - 2x - 4 - 1}{4}$

$= \frac{-4}{4}$

$= \mathbf{-1}$ ◄

*This is an **expression**:*

*keep the value the same; do **not** multiply it!*

All terms need to be written over a common (the same) denominator.

5.10 $\frac{x - y}{y + x} \times \frac{(x+y)^2}{x - y}$

$= \frac{\cancel{(x-y)}}{\cancel{(x+y)}} \times \frac{(x+y)^{\cancel{2}}}{\cancel{(x-y)}}$

$= \frac{x + y}{1}$

$= \mathbf{x + y}$ ◄

Compare to: $\frac{a}{b} \times \frac{b^2}{a}$

$= b$ ◄

5.11 $\frac{x-2}{2x} - \frac{x-3}{3x}$ . . . *Do not multiply!*

$= \frac{3(x-2) - 2(x-3)}{6x}$ . . . *NB: Brackets!*

$= \frac{3x - 6 - 2x + 6}{6x}$ . . . *Keep the denominator, 6x!*

$= \frac{x}{6x}$

$= \mathbf{\frac{1}{6}}$ ◄

NB:

It is only when working with fractions **IN EQUATIONS** (Page Q6, Question 4), that you **logically** multiply both sides of the equation!

5.12 $\frac{4x^2}{2a^2} \div \frac{4x}{2a^2}$

$= \frac{4x^2}{2a^2} \times \frac{2a^2}{4x}$

$= \boldsymbol{x}$ ◄

5.13 $\frac{15x^2y^3 + 9x^2y^3}{8x^2y^3}$ . . . *LIKE TERMS* ☺

$= \frac{24x^2y^3}{8x^2y^3}$

$= \mathbf{3}$ ◄

5.14 $\frac{5a^2b}{3ab} \div \frac{20a^3b}{27}$

$= \frac{5a}{3} \times \frac{27}{20a^3b}$

$= \mathbf{\frac{9}{4a^2b}}$ ◄

5.15 $\dfrac{5}{b} - \dfrac{4}{a} - \dfrac{a-b}{ab}$ . . . *Do not multiply!*

$= \dfrac{5a - 4b - (a - b)}{ab}$ . . . *NB: Brackets!*

$= \dfrac{5a - 4b - a + b}{ab}$ . . . *Keep the denominator, ab!*

$= \mathbf{\dfrac{4a - 3b}{ab}}$ ➤

5.16 $\dfrac{3a^{-2}b \times 24b^{-1}a^{-1}}{9a^{-4}b^{-3}}$

$= \dfrac{72a^{-3}}{9a^{-4}b^{-3}}$

$= \mathbf{8ab^3}$ ➤ . . . $\dfrac{a^{-3}}{a^{-4}} = a^{-3-(-4)} = a^1$

& $\dfrac{1}{b^{-3}} = b^3$

5.17 $\dfrac{x^2}{2} + \dfrac{2x^2}{3} - \dfrac{7x^2}{6}$

$= \dfrac{3x^2 + 2(2x^2) - 7x^2}{6}$ . . . *Write all 3 terms over a common denominator, 6.*

$= \dfrac{3x^2 + 4x^2 - 7x^2}{6}$ . . . *Do not multiply (by 6)!*

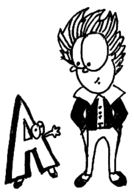
$= \dfrac{0}{6}$

$= \mathbf{0}$ ➤

5.18 $\dfrac{6x^2}{7xy} \times \dfrac{3y^3}{2x}$

$= \dfrac{6x}{7y} \times \dfrac{3y^3}{2x}$

$= \mathbf{\dfrac{9y^2}{7}}$ ➤



Square roots and cube roots

5.19 $\sqrt{225x^4} - \sqrt[3]{125x^6}$

$= 15x^2 - 5x^2$

$= \mathbf{10x^2}$ ➤

$$\begin{array}{r|l} 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\therefore\ 125 = 5^3$

$\therefore\ 225 = 3^2 \times 5^2$ $\qquad$ $\therefore\ \sqrt[3]{125} = 5$

$\therefore\ \sqrt{225} = 3 \times 5$

5.20 $\sqrt{16x^{16} \times 25x^4}$ $\qquad$ OR: $= \sqrt{400x^{20}}$

$= \sqrt{16x^{16}} \cdot \sqrt{25x^4}$ $\qquad$ $= \sqrt{20x^{10}}$

$= 4x^8 \cdot 5x^2$

$= \mathbf{20x^{10}}$ ➤

5.21 $\sqrt[3]{27x^{27}}$ . . . $3 \times 3 \times 3 = 27$; $x^9 \times x^9 \times x^9 = x^{27}$

$= \mathbf{3x^9}$ ➤

5.22 $\sqrt{16a^2 + 9a^2}$ . . . *1st add LIKE TERMS*

$= \sqrt{25a^2}$

$= \mathbf{5a}$ ➤

NB: $\sqrt{16a^2 + 9a^2} \neq 4a + 3a$!

6.1 $4ab(5a^2b^2 + 2ab - 3)$

$= \mathbf{20a^3b^3 + 8a^2b^2 - 12ab}$ ➤

6.2 $3a^2bc^2(3a^2 - 4b - c)$

$= \mathbf{9a^4bc^2 - 12a^2b^2c^2 - 3a^2bc^3}$ ➤

Distributive Property:

$a(b + c)$
$= ab + ac$

and

$a(b - c)$
$= ab - ac$

6.3 $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$

$= x^2 + 7x + 10$

6.4 $(x - 2)(x - 3) = x^2 - 3x - 2x + 6$

$= x^2 - 5x + 6$

6.5 $(x + 7)(x - 1) = x^2 - x + 7x - 7$

$= x^2 + 6x - 7$

6.6 $(2x - 3)(x + 1)$

$= 2x^2 + 2x - 3x - 3$

$= \mathbf{2x^2 - x - 3}$ ➤

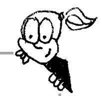
6.7 $x(x + 2) - (x - 1)(x - 3)$

$= x^2 + 2x - (x^2 - 3x - x + 3)$

$= x^2 + 2x - (x^2 - 4x + 3)$

$= x^2 + 2x - x^2 + 4x - 3$

$= \mathbf{6x - 3}$ ➤



6.8 $(x - 3)^2 - x(x + 4)$

$= x^2 - 6x + 9 - x^2 - 4x$

$= \mathbf{-10x + 9}$ ➤

$(x - 3)^2$

$= (x - 3)(x - 3)$

$= x^2 \mathbf{- 3x - 3x} + 9$

$= x^2 \mathbf{- 6x} + 9$

Perfect Square Trinomial

6.9 $(2x-1)^2 - (x+1)(x-1)$

$= (2x-1)(2x-1) - (x^2-1)$

$= 4x^2 - 4x + 1 - x^2 + 1$

$= \mathbf{3x^2 - 4x + 2}$ ➤

Perfect Square Trinomial →

$(2x-1)^2 = (2x-1)(2x-1)$
$= 4x^2 \mathbf{- 2x - 2x} + 1$
$= 4x^2 \mathbf{- 4x} + 1$

Difference between Squares →

& $(x+1)(x-1) = x^2 - x + x - 1$
$= x^2 - 1$

6.10 $2(x+2)^2 - (2x-1)(x+2)$

$= 2(x+2)(x+2) - (2x^2 + 4x - x - 2)$

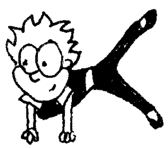
$= 2(x^2 + 4x + 4) - (2x^2 + 3x - 2)$

$= 2x^2 + 8x + 8 - 2x^2 - 3x + 2$

$= \mathbf{5x + 10}$ ➤

Perfect Square Trinomial →

$(x+2)^2$
$= (x+2)(x+2)$
$= x^2 \mathbf{+ 2x + 2x} + 4$
$= x^2 \mathbf{+ 4x} + 4$



7.1 ➨ 7.4: Perfect Square Trinomials

7.1 $(x+5)^2 = (x+5)(x+5)$

$= x^2 \mathbf{+ 5x + 5x} + 25$

$= \mathbf{x^2 + 10x + 25}$ ➤

7.2 $(p-3)^2 = (p-3)(p-3)$

$= p^2 \mathbf{- 3p - 3p} + 9$

$= \mathbf{p^2 - 6p + 9}$ ➤

7.3 $(2a+3)^2 = (2a+3)(2a+3)$

$= 4a^2 \mathbf{+ 6a + 6a} + 9$

$= \mathbf{4a^2 + 12a + 9}$ ➤

7.4 $(4x-1)^2 = (4x-1)(4x-1)$

$= 16x^2 \mathbf{- 4x - 4x} + 1$

$= \mathbf{16x^2 - 8x + 1}$ ➤

7.5 ➨ 7.8: Difference between Squares

7.5 $(x+5)(x-5) = \mathbf{x^2 - 5x + 5x - 25} = \mathbf{x^2 - 25}$

7.6 $(p-3)(p+3) = \mathbf{p^2 + 3p - 3p - 9} = \mathbf{p^2 - 9}$

7.7 $(2a+3)(2a-3) = \mathbf{4a^2 - 6a + 6a - 9} = \mathbf{4a^2 - 9}$

7.8 $(4x-1)(4x+1) = \mathbf{16x^2 + 4x - 4x - 1} = \mathbf{16x^2 - 1}$

7.9 ➨ 7.12: Observe **how** the **trinomial** is obtained in each case.

7.9 $(x+3)(x+4) = x^2 + 4x + 3x + 12 = \mathbf{x^2 + 7x + 12}$ ➤

7.10 $(x-3)(x-4) = x^2 - 4x - 3x + 12 = \mathbf{x^2 - 7x + 12}$ ➤

7.11 $(x+3)(x-4) = x^2 - 4x + 3x - 12 = \mathbf{x^2 - x - 12}$ ➤

7.12 $(x-3)(x+4) = x^2 + 4x - 3x - 12 = \mathbf{x^2 + x - 12}$ ➤

8.1 **B** ➤ . . . $-(-2)^2 - (2(-2) - 1)$

$= -(4) - (-4 - 1)$

$= -4 - (-5)$

$= -4 + 5$

$= 1$

8.2 **C** ➤

8.3 **D** ➤ . . . $\dfrac{x}{y} - \dfrac{1^{\times y}}{1_{\times y}} = \dfrac{x - y}{y}$ . . . *Write the terms over the same denominator.*

8.4 **D** ➤ . . . $\left(\dfrac{x}{3} - 3y\right)\left(\dfrac{x}{3} + 3y\right)$

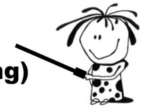
$= \left(\dfrac{x}{3}\right)^2 - (3y)^2$ . . . *difference of squares*

$= \dfrac{x^2}{9} - 9y^2$

FACTORISATION

1. Common Factor

Check each answer by multiplying back (to the beginning)



1.1 $8p^3 + 4p^2$
$= \mathbf{4p^2(2p + 1)}$

1.2 $10t^2 - 5t$
$= \mathbf{5t(2t - 1)}$

1.3 $3x^2y - 9xy^2 + 12x^3y^3$
$= \mathbf{3xy(x - 3y + 4x^2y^2)}$

1.4 $2p^2 + 2$
$= \mathbf{2(p^2 + 1)}$

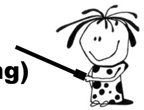
1.5 $2(x + y) + a(x + y)$
$= \mathbf{(x + y)(2 + a)}$

1.6 $2(x + y) - t(x + y)$
$= \mathbf{(x + y)(2 - t)}$

1.7 $tx - ty - 2x + 2y$
$= (tx - ty) - (2x - 2y)$
$= t(x - y) - 2(x - y)$
$= \mathbf{(x - y)(t - 2)}$

2. Difference between Squares

Check each answer by multiplying back (to the beginning)



2.1 $4x^2 - y^2$
$= \mathbf{(2x + y)(2x - y)}$

2.2 $4x^2 - 4y^2$
$= 4(x^2 - y^2)$
$= \mathbf{4(x + y)(x - y)}$

Compare Question 2.1 and Question 2.2!
*In Question 2.1: 4 is **not** a common factor.*
*In Question 2.2: 4 **is** a common factor.*

2.3 $81 - 100a^2$
$= \mathbf{(9 + 10a)(9 - 10a)}$

2.4 $9p^2 - 36q^2$
$= 9(p^2 - 4q^2)$
$= \mathbf{9(p + 2q)(p - 2q)}$

2.5 $7x^2 - 28$
$= 7(x^2 - 4)$
$= \mathbf{7(x + 2)(x - 2)}$

3. Trinomials

3 TERMS

Observe these PRODUCTS:

- **Perfect Squares ➡ Perfect Square TRINOMIALS**

$(x+3)^2 = (x+3)(x+3) = x^2 \mathbf{+ 3x + 3x} + 9 = x^2 \mathbf{+ 6x} + 9$
& $(x-3)^2 = (x-3)(x-3) = x^2 \mathbf{- 3x - 3x} + 9 = x^2 \mathbf{- 6x} + 9$
$\therefore\ x^2 + 6x + 9 = \mathbf{(x + 3)^2}$
& $x^2 - 6x + 9 = \mathbf{(x - 3)^2}$

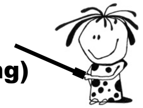
$(a+b)^2 = (a+b)(a+b) = a^2 \mathbf{+ ab + ab} + b^2 = a^2 \mathbf{+ 2ab} + b^2$
& $(a-b)^2 = (a-b)(a-b) = a^2 \mathbf{- ab - ab} + b^2 = a^2 \mathbf{- 2ab} + b^2$
$\therefore\ a^2 + 2ab + b^2 = \mathbf{(a + b)^2}$
& $a^2 - 2ab + b^2 = \mathbf{(a - b)^2}$

- **Other products ➡ TRINOMIALS**

$(x + 2)(x + 3) = \mathbf{x^2 + 2x + 3x + 6} = \mathbf{x^2 + 5x + 6}$
$(x - 2)(x - 3) = \mathbf{x^2 - 2x - 3x + 6} = \mathbf{x^2 - 5x + 6}$
$(x + 2)(x - 3) = \mathbf{x^2 + 2x - 3x - 6} = \mathbf{x^2 - x - 6}$
$(x - 2)(x + 3) = \mathbf{x^2 - 2x + 3x - 6} = \mathbf{x^2 + x - 6}$

Observe the results above to understand factorising trinomials

Check each answer by multiplying back (to the beginning)



3.1 $a^2 + 8a + 16 = (a + 4)(a + 4) = \mathbf{(a + 4)^2}$

3.2 $p^2 - 10p + 25 = (p - 5)(p - 5) = \mathbf{(p - 5)^2}$

3.3 $x^2 + 5x + 6 = \mathbf{(x + 3)(x + 2)}$ . . . $\begin{array}{l} 1 \quad +3 \\ 1 \quad +2 \end{array}$ $\begin{array}{r} +3 \\ +2 \\ \hline +5 \end{array}$

3.4 $x^2 - 5x + 6 = \mathbf{(x - 3)(x - 2)}$ . . . $\begin{array}{l} 1 \quad -3 \\ 1 \quad -2 \end{array}$ $\begin{array}{r} -3 \\ -2 \\ \hline -5 \end{array}$

3.5 $x^2 + x - 6 = \mathbf{(x + 3)(x - 2)}$ . . . $\begin{array}{l} 1 \quad +3 \\ 1 \quad -2 \end{array}$ $\begin{array}{r} +3 \\ -2 \\ \hline +1 \end{array}$

3.6 $x^2 - x - 6 = \mathbf{(x - 3)(x + 2)}$ . . . $\begin{array}{l} 1 \quad -3 \\ 1 \quad +2 \end{array}$ $\begin{array}{r} -3 \\ +2 \\ \hline -1 \end{array}$

3.7 $x^2 - 11x + 18 = \mathbf{(x - 9)(x - 2)}$ . . . $\begin{array}{l} 1 \quad -9 \\ 1 \quad -2 \end{array}$ $\begin{array}{r} -9 \\ -2 \\ \hline -11 \end{array}$

3.8 $x^2 + 11x + 18 = \mathbf{(x + 9)(x + 2)}$ . . . $\begin{array}{l} 1 \quad +9 \\ 1 \quad +2 \end{array}$ $\begin{array}{r} +9 \\ +2 \\ \hline +11 \end{array}$

3.9 $x^2 - 7x - 18 = \mathbf{(x - 9)(x + 2)}$ . . . $\begin{array}{l} 1 \quad -9 \\ 1 \quad +2 \end{array}$ $\begin{array}{r} -9 \\ +2 \\ \hline -7 \end{array}$

3.10 $x^2 + 7x - 18 = \mathbf{(x + 9)(x - 2)}$ ➤ . . . $\begin{array}{c} 1 \;\; +9 \\ 1 \;\; -2 \end{array}$ $\quad \begin{array}{r} +9 \\ -2 \\ \hline +7 \end{array}$

3.11 $x^2 + 9x + 18 = \mathbf{(x + 6)(x + 3)}$ ➤ . . . $\begin{array}{c} 1 \;\; +6 \\ 1 \;\; +3 \end{array}$ $\quad \begin{array}{r} +6 \\ +3 \\ \hline +9 \end{array}$

3.12 $x^2 - 9x + 18 = \mathbf{(x - 6)(x - 3)}$ ➤ . . . $\begin{array}{c} 1 \;\; -6 \\ 1 \;\; -3 \end{array}$ $\quad \begin{array}{r} -6 \\ -3 \\ \hline -9 \end{array}$

3.13 $x^2 + 3x - 18 = \mathbf{(x + 6)(x - 3)}$ ➤ . . . $\begin{array}{c} 1 \;\; +6 \\ 1 \;\; -3 \end{array}$ $\quad \begin{array}{r} +6 \\ -3 \\ \hline +3 \end{array}$

3.14 $x^2 - 3x - 18 = \mathbf{(x - 6)(x + 3)}$ ➤ . . . $\begin{array}{c} 1 \;\; -6 \\ 1 \;\; +3 \end{array}$ $\quad \begin{array}{r} -6 \\ +3 \\ \hline -3 \end{array}$

FACTORISATION

There are **3 TYPES** of factorization :

❶ **Common Factor (CF):** Always try this first !

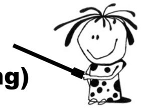
❷ **Difference between Squares (DbS):** 2 terms

❸ **Trinomials:** 3 terms

RECOGNISE THESE !

Mixed Factorisation

Check each answer by multiplying back (to the beginning)



4.1 $3a^3 - 9a^2 - 6a$

$= \mathbf{3a(a^2 - 3a - 2)}$ ➤ . . . *note : this does not factorise further*

4.2 $2a^2 - 18a + 36$

$= 2(a^2 - 9a + 18)$ $\qquad \begin{array}{c} 1 \;\; -6 \\ 1 \;\; -3 \end{array}$ $\quad \begin{array}{r} -6 \\ -3 \\ \hline -9 \end{array}$

$= \mathbf{2(a - 6)(a - 3)}$ ➤

4.3 $4(a + b) - x^2(a + b)$

$= (a + b)(4 - x^2)$ . . . *always check to see if you can factorise further*

$= \mathbf{(a + b)(2 + x)(2 - x)}$ ➤

4.4 $6x^3(a - b) + x(b - a)$

$= 6x^3(a - b) - x(a - b)$ . . . *switchround*

$= (a - b)(6x^3 - x)$ . . . *the 'new' factor can factorise further*

$= (a - b) \,.\, x(6x^2 - 1)$

$= \mathbf{x(a - b)(6x^2 - 1)}$ ➤

4.5 $6a^3 - 12a^2 + 18a$

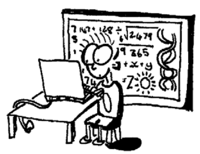
$= \mathbf{6a(a^2 - 2a + 3)}$ ➤ . . . *note : this does not factorise further*

Use factorisation to simplify the following fractions

5.1 $\dfrac{x^2 - 1}{3x + 3}$ . . . **DbS** . . . **CF**

$= \dfrac{\cancel{(x + 1)}(x - 1)}{3\cancel{(x + 1)}}$

$= \mathbf{\dfrac{x - 1}{3}}$ ➤



5.2 $\dfrac{x^2 - 4x}{x^2 - 2x - 8}$ . . . **CF** . . . **Trinomial** . . . $\begin{array}{c} 1 \;\; -4 \\ 1 \;\; +2 \end{array}$ $\quad \begin{array}{r} -4 \\ +2 \\ \hline -2 \end{array}$

$= \dfrac{x\cancel{(x - 4)}}{\cancel{(x - 4)}(x + 2)}$

$= \mathbf{\dfrac{x}{x + 2}}$ ➤

5.3 $\dfrac{3a - 6b}{4b - 2a}$

$= \dfrac{3(a - 2b)}{2(2b - a)}$ . . . **Common Factors**

$= \dfrac{3\cancel{(a - 2b)}}{-2\cancel{(a - 2b)}}$ . . . *2b − a = −(a − 2b)*

$= -\mathbf{\dfrac{3}{2}}$ ➤

5.4 $\dfrac{2x^2 - 8}{3x - 12} \times \dfrac{x^2 - 4x}{x - 2}$

$= \dfrac{2(x^2 - 4)}{3\cancel{(x - 4)}} \times \dfrac{x\cancel{(x - 4)}}{x - 2}$

$= \dfrac{2(x + 2)\cancel{(x - 2)}}{3\cancel{(x - 2)}}$

$= \mathbf{\dfrac{2(x + 2)}{3}}$ ➤

Don't be frightened by the look of these fractions !

Just focus on factorising where possible ; then cancel the factors where possible.

5.5 $\dfrac{x^2 + 2x}{x^3 - 2x} \div \dfrac{x^2 - 4}{x - 2}$

$= \dfrac{x^2 + 2x}{x^3 - 2x} \times \dfrac{x - 2}{x^2 - 4}$ . . . *note the 'flipped' fraction !*

$= \dfrac{\cancel{x}\cancel{(x + 2)}}{\cancel{x}(x^2 - 2)} \times \dfrac{\cancel{(x - 2)}}{\cancel{(x + 2)}\cancel{(x - 2)}}$

$= \mathbf{\dfrac{1}{x^2 - 2}}$ ➤

ALGEBRAIC EQUATIONS
(Linear and Quadratic)

1.1 **B** ◂ . . . *If 3 is a root, then $x = 3$ will make the equation true*

i.e. $3^2 + 3 + t = 0$

$\therefore\ 9 + 3 + t = 0$

$\therefore\ t = -12$

1.2 **D** ◂ . . . $2p + 12 = 58$

$\therefore\ 2p = 46$

$\therefore\ p = 23$

OR: $? + 12 = 58$
Answer: 46
So, $2p = 46$
$\therefore\ 2 \times ? = 46$
$\therefore\ p = 23$

1.3 **B** ◂ . . . ***If*** $(x-1)(x+2) = 0$**,**

then $x - 1 = 0$ ***or*** $x + 2 = 0$

$\therefore\ x = 1$ $\quad\therefore\ x = -2$

1.4 **D** ◂ . . . $\dfrac{3x}{2} = -6$

$\therefore\ 3x = -12$

$x = -4$

OR: $\dfrac{?}{2} = -6$
Answer: -12
$\therefore\ 3x = -12$
So, $3 \times ? = -12$
$\therefore\ x = -4$

1.5 **B** ◂ . . . *The product of a number, x, and 6 equals* $x \times 6 = 6x$

2.1 $x + 5 = 2$ . . . *'5 more than a number is 2'*

$\therefore\ \boldsymbol{x = -3}$ ◂

2.2 $x - 3 = -4$ . . . *'3 less than a number is -4'*

$\therefore\ \boldsymbol{x = -1}$ ◂

2.3 $2x = 12$ . . . *'double a number is 12'*

$\therefore\ \boldsymbol{x = 6}$ ◂

2.4 $\dfrac{x}{5} = 6$ . . . *'a fifth of a number is 6'*

$\therefore\ \boldsymbol{x = 30}$ ◂

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

3.1 $3x - 1 = 5$

$\therefore\ 3x = 6$

$\therefore\ \boldsymbol{x = 2}$ ◂

[Check: LHS $= 3x - 1 = 3(2) - 1 = 5 =$ **RHS** ✓**]**

3.2 $2(x+1) = 10$

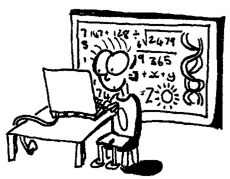
$\therefore\ 2x + 2 = 10$

$\therefore\ 2x = 8$

$\therefore\ \boldsymbol{x = 4}$ ◂

. . . OR: $x + 1 = 5$
$\therefore\ x = 4$ ◂

[Check: LHS $= 2(4+1) = 2 \times 5 = 10 =$ **RHS** ✓**]**



3.3 $8x + 3 = 3x - 22$

$\therefore\ 8x - 3x = -22 - 3$ . . . *Subtract $3x$ & Subtract 3*

$\therefore\ 5x = -25$

$\therefore\ \boldsymbol{x = -5}$ ◂ . . . $\left(\div 5,\ \text{OR: } 5 \times \boldsymbol{?} = -25\right)$

Check: **LHS** $= 8(-5) + 3 = -40 + 3 = -37$

& **RHS** $= 3(-5) - 22 = -15 - 22 = -37$

$\therefore$ **LHS = RHS** ✓ $\therefore$ The answer is correct.

i.e. The equation is 'true' for $x = -5$.

We say that -5 is the **root** (or solution) of the equation.

3.4 $3(x+6) = 12$

$\therefore\ 3x + 18 = 12$ . . . OR: $x + 6 = 4$ $\quad\therefore\ x = -2$ ◂

$\therefore\ 3x = -6$

$\therefore\ \boldsymbol{x = -2}$ ◂

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

3.5 $2x - 5 = 5x + 16$ . . . *Add 5 & Subtract $5x$*

$\therefore\ 2x - 5x = 16 + 5$

$\therefore\ -3x = 21$

$\therefore\ \boldsymbol{x = -7}$ ◂ . . . *Divide by -3*

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

3.6 $x^3 + x^3 = 2$

$\therefore\ 2x^3 = 2$ . . . *LIKE TERMS*

$\therefore\ x^3 = 1$ . . . *divide by 2*

$\therefore\ \boldsymbol{x = 1}$ ◂ . . . *take cube root*

Check your answer by substituting in the given equation to see if it is 'true' for the value of x.

Equations including fractions

> ***NOTE:*** *In* ***EXPRESSIONS****, the previous section, we did* ***not*** *multiply. We kept the denominator!*
>
> *Now, in* ***EQUATIONS****, we do multiply, applying logic!!! – 'what we do to the left hand side* ***(LHS)*** *of an equation, we also do to the right hand side* ***(RHS)****'.*

4.1 $\quad \frac{x-2}{4} + \frac{2x+1}{3} = \frac{5}{3}$

NB: Brackets!

$$\begin{aligned}
\textbf{x12)} \quad \therefore 3(x-2) + 4(2x+1) &= 4(5) \\
\therefore 3x - 6 + 8x + 4 &= 20 \\
\therefore 11x - 2 &= 20 \\
\therefore 11x &= 22 \\
\therefore \boldsymbol{x} &= \mathbf{2}
\end{aligned}$$

Check your answer!

The LCM of the denominators is 12.
The logic: 12 × the ***LHS*** *= 12 × the* ***RHS***

4.2 $\quad \frac{x+2}{3} - \frac{x-3}{4} = 0$

NB: Brackets!

$$\begin{aligned}
\textbf{x12)} \quad \therefore 4(x+2) - 3(x-3) &= 0 \\
\therefore 4x + 8 - 3x + 9 &= 0 \\
\therefore x + 17 &= 0 \\
\therefore \boldsymbol{x} &= \mathbf{-17}
\end{aligned}$$

Check your answer!

4.3 $\quad \frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-1}{2}$

NB: Brackets!

$$\begin{aligned}
\textbf{x6)} \quad 3(2x-3) - 2(x+1) &= 3(3x-1) \\
\therefore 6x - 9 - 2x - 2 &= 9x - 3 \\
\therefore 4x - 11 &= 9x - 3 \\
\therefore 4x - 9x &= -3 + 11 \\
\therefore -5x &= 8 \\
\therefore \boldsymbol{x} &= \mathbf{-\frac{8}{5}}
\end{aligned}$$

6 times the ***LHS*** *= 6 times the* ***RHS***

Check your answer!

4.4 $\quad \frac{x}{1} - \frac{x-1}{2} = \frac{3}{1}$

$$\begin{aligned}
\textbf{x2)} \quad \therefore 2x - (x-1) &= 2(3) \\
\therefore 2x - x + 1 &= 6 \\
\therefore x + 1 &= 6 \\
\therefore \boldsymbol{x} &= \mathbf{5}
\end{aligned}$$

NB: Brackets!

2 times the ***LHS*** *= 2 times the* ***RHS***

Check your answer!

4.5 $\quad \frac{x+1}{3} - \frac{x-1}{6} = \frac{1}{1}$

$$\begin{aligned}
\textbf{x6)} \quad \therefore 2(x+1) - (x-1) &= 6(1) \\
\therefore 2x + 2 - x + 1 &= 6 \\
\therefore x + 3 &= 6 \\
\therefore \boldsymbol{x} &= \mathbf{3}
\end{aligned}$$

NB: Brackets!

6 times the ***LHS*** *= 6 times the* ***RHS***

Check your answer!

> **Compare the position of the = signs**
>
> ***In Algebraic Expressions*** *(in the previous section):*
> *The* ***= signs*** *are down the* ***left***
>
> ***In Algebraic Equations*** *(Q4.1 to 4.5 above):*
> *The* ***= signs*** *are in the* ***middle***
> *and the* $\therefore$ ***signs*** *are on the* ***left***

Quadratic Equations

> **The LOGIC:**
> **If the product of 2 numbers = 0, then either one or the other number must = 0.**

5.1 $\quad (x-3)(x+4) = 0$ . . . *the product of $x-3$ and $x-4$ equals 0, so:*

Either $x - 3 = 0$ **or** $x + 4 = 0$

$\therefore \boldsymbol{x} = \mathbf{3}$ $\quad\quad \therefore \boldsymbol{x} = \mathbf{-4}$

Check: If $x = 3$:
LHS $= (3-3)(3+4) = 0(7) = 0 =$ **RHS** ✓

If $x = -4$:
LHS $= (-4-3)(-4+4) = (-7) \times 0 = 0 =$ **RHS** ✓

$\therefore$ Both answers are correct

5.2 $\quad x^2 - 5x - 6 = 0$ . . . *Factorise the* ***trinomial*** *so that you have a product*

$\therefore (x-6)(x+1) = 0$

$\therefore$ **Either** $x - 6 = 0$ **or** $x + 1 = 0$

$\therefore \boldsymbol{x} = \mathbf{6}$ $\quad\quad \boldsymbol{x} = \mathbf{-1}$

1×-6, $1 + 1$: $\quad -6$, $+1$, -5

Check your answers!

5.3 $\quad x^2 - 1 = 0$ ... *Factorise!* ***(Difference between squares)***

$\therefore (x+1)(x-1) = 0$

$\therefore$ **Either** $x + 1 = 0$ **or** $x - 1 = 0$

$\therefore \boldsymbol{x} = \mathbf{-1}$ $\quad\quad \boldsymbol{x} = \mathbf{1}$

OR: $x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

Check your answers!

5.4 $x^2 - 2x = 0$ . . . *Factorise !* ***(Common Factor)***

$\therefore\ x(x-2) = 0$

$\therefore$ **Either** $\boldsymbol{x = 0}$ ➤ **or** $x - 2 = 0$

$\therefore\ \boldsymbol{x = 2}$ ➤

Check your answers!

In Questions 6.1 and 6.2 :

$(x-2)^2$
$= (x-2)(x-2)$
$= x^2 \mathbf{- 2x - 2x} + 4$
$= x^2 \mathbf{- 4x} + 4$

In Question 6.3 :

$(x+3)^2$
$= (x+3)(x+3)$
$= x^2 \mathbf{+ 3x + 3x} + 9$
$= x^2 \mathbf{+ 6x} + 9$

Now, the equations . . .

6.1 $2(x-2)^2 = (2x-1)(x-3)$

$\therefore\ 2(x-2)(x-2) = 2x^2 - 6x - x + 3$

$\therefore\ 2(x^2 - 4x + 4) = 2x^2 - 7x + 3$

$\therefore\ 2x^2 - 8x + 8 = 2x^2 - 7x + 3$

$\therefore\ -8x + 7x = 3 - 8$ . . . *the two $2x^2$ terms cancel each other and the equation becomes **linear** (no longer **quadratic**).*

$\therefore\ -x = -5$

$\therefore\ \boldsymbol{x = 5}$ ➤

Check your answer!

6.2 $(x-2)^2 + 3x - 2 = (x+3)^2$

$\therefore\ (x-2)(x-2) + 3x - 2 = (x+3)(x+3)$

$\therefore\ x^2 - 4x + 4 + 3x - 2 = x^2 + 6x + 9$

$\therefore\ -x + 2 = 6x + 9$ . . . *the two $2x^2$ terms cancel each other and the equation becomes **linear**.*

$\therefore\ -x - 6x = 9 - 2$

$\therefore\ -7x = 7$

$\therefore\ \boldsymbol{x = -1}$ ➤

Check your answer!

6.3 $(x-3)^2 = 16$

$(x-3)(x-3) = 16$

$\therefore\ x^2 - 6x + 9 - 16 = 0$

$\therefore\ x^2 - 6x - 7 = \mathbf{0}$ ⬅ *Remember the logic ? The product must **= 0** !*

$\therefore\ (x-7)(x+1) = 0$ ⬅ *The **trinomial** is factorised*

The logic :

$(x-7)$ times $(x+1)$ equals 0, so . . . .

Either $x - 7$ equals 0 **or** $x + 1$ equals 0

i.e. **Either** $x - 7 = 0$ **or** $x + 1 = 0$

$\therefore\ \boldsymbol{x = 7}$ ➤ **or** $\boldsymbol{x = -1}$ ➤

Check your answers!

Other . . .

Note :
Solving equations requires ***reversing operations***

+ ⟷ -	**x ⟷ ÷**	**powers ⟷ roots**
$x \mathbf{+3} = 8$ $\therefore\ x + 3 \mathbf{-3} = 8 \mathbf{-3}$ $\therefore\ x = 5$ ➤	$\mathbf{3}x = 12$ $\therefore\ \frac{3x}{\mathbf{3}} = \frac{12}{\mathbf{3}}$ $\therefore\ x = 4$ ➤	$x^{\mathbf{3}} = 8$ Take $\sqrt[3]{\ }$ on both sides $\therefore\ x = 2$ ➤ ***Note :*** $x^{\mathbf{2}} = 9$ $\therefore\ x = \pm 3$ *If the power is even, there are 2 roots !*
$x \mathbf{-3} = 8$ $\therefore\ x - 3 \mathbf{+3} = 8 \mathbf{+3}$ $\therefore\ x = 11$ ➤	$\frac{x}{\mathbf{3}} = 12$ $\therefore\ \frac{x}{3}\ \mathbf{x\,3} = 12\ \mathbf{x\,3}$ $\therefore\ x = 36$ ➤	$\sqrt{x} = 5$ **Square** both sides $(\sqrt{x})^2 = 5^2$ $\therefore\ x = 25$ ➤

ALWAYS CHECK YOUR ANSWER !

So, solve for x : (a) $\sqrt{x} = 2$

Square both sides

$\therefore\ x = 4$ ➤

(b) $\sqrt{\sqrt{x}} = 2$

Square both sides

$\therefore\ \sqrt{x} = 4$

Square both sides again

$\therefore\ x = 16$ ➤

Now see Q7.1

7.1 $$\sqrt{\sqrt{\sqrt{x}}} = 2$$

Square both sides $$\therefore \left(\sqrt{\sqrt{\sqrt{x}}}\right)^2 = (2)^2$$

$$\therefore \sqrt{\sqrt{x}} = 4$$

Square both sides again $$\therefore \left(\sqrt{\sqrt{x}}\right)^2 = (4)^2$$

$$\therefore \sqrt{x} = 16$$

Square both sides again ! $$\therefore \left(\sqrt{x}\right)^2 = (16)^2$$

$$\therefore \boldsymbol{x} = \mathbf{256}$$

Check your answer!

7.2 $$\sqrt{\frac{1}{\sqrt{x}}} = 2$$

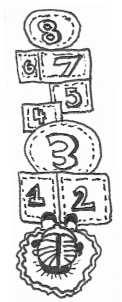
$$\therefore \left(\sqrt{\frac{1}{\sqrt{x}}}\right)^2 = (2)^2$$

$$\therefore \frac{1}{\sqrt{x}} = 4$$

$$\therefore \left(\frac{1}{\sqrt{x}}\right)^2 = (4)^2$$

$$\therefore \frac{1}{x} = \frac{16}{1}$$

$$\therefore \boldsymbol{x} = \mathbf{\frac{1}{16}}$$



Check your answer!

NOTES

GRAPHS

1.1 **B** ◄ . . . $f(x) = 2x + 4$:

- *positive gradient of* $\frac{2}{1}$ . . . $\mathbf{2}x$,
- *y–int of 4* . . . $y = 4$ *when* $x = 0$

If a point lies on a line, then the **equation** of the graph will be **true** for its coordinates. *(See Question 1.2)*

1.2 **D** ◄ . . . *The equation is* $y = x$, *so* x *and y will have to be equal (i.e. the coordinates must have the same value)*

1.3 **B** ◄ . . . $\frac{d}{6} = \frac{2}{3}$ $\therefore$ $d = 4$ . . . *equivalent fractions*

Very important to know:

On the **Y-axis**, the $\boldsymbol{x}$-coordinate is (always) **0** *(See Question 1.4)*

On the **X-axis**, the **y**-coordinate is (always) **0** *(See Question 1.5)*

1.4 **C** ◄ . . . *Substitute* $x = 0$; *then*

$$\text{y–intercept: } 4(0) + 2y = 12$$
$$\therefore\ 2y = 12$$
$$\therefore\ y = 6$$

So, the point on the y–axis is (0; 6)

1.5 **B** ◄ . . . *Substitute* $y = 0$; *then*

$$\text{x–intercept: } 3(0) + 2x + 1 = 0$$
$$\therefore\ 2x = -1$$
$$\therefore\ x = -\frac{1}{2}$$

$\therefore \left(-\frac{1}{2};\ 0\right)$ . . . *the coordinates of the x–intercept*

2. P is the intersection of the lines $y = x$ and $y = 3$ and so at point P, both these equations must 'be true'.

So, y must equal x and y must equal 3. $\therefore$ $y = x = 3$

$\therefore$ **P(3; 3)** ◄

3.1 $y = 3x - 5$

x	-2	-1	0	1
y	-11	-8	-5	-2

$y = 3(-2) - 5 = -11$

$y = 3(-1) - 5 = -8$

$y = 3(0) - 5 = -5$

$y = 3(1) - 5 = -2$

We substitute the values of x into the equation to find y.

3.2 $y = -\frac{2}{3}x - 1$

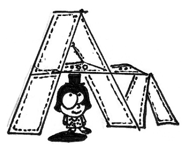
x	-3	-1	0	1
y	1	$-\frac{1}{3}$	-1	$-\frac{5}{3}$

$$y = -\frac{2}{3}\left(-\frac{3}{1}\right) - 1 = 2 - 1 = 1$$

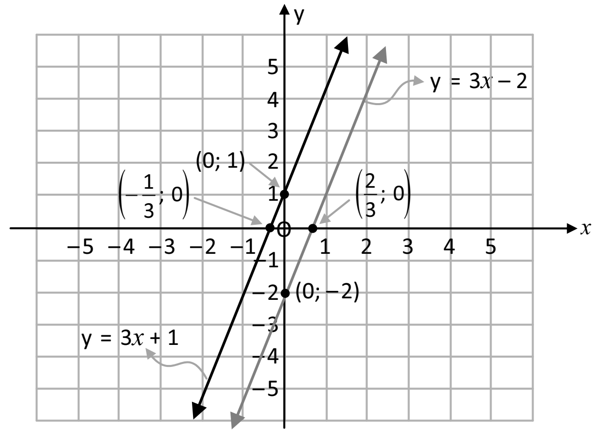
$$y = -\frac{2}{3}(-1) - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$y = -\frac{2}{3}(0) - 1 = -1$$

$$y = -\frac{2}{3}(1) - 1 = -\frac{2}{3} - 1 = -\frac{5}{3}$$



4.1



To find the points where the graphs cut the axes:

y = 3x – 2:

For the **Y-intercept**, substitute $\boldsymbol{x = 0}$

$\therefore\ y = 3(0) - 2$
$= -2$

$\therefore$ The graph cuts the **y–axis** at -2.

The **point** is (**0**; 2)

For the **X-intercept**, substitute **y = 0**

$\therefore\ 0 = 3x - 2$

$\therefore\ 3x = 2$

$\therefore\ x = \frac{2}{3}$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{2}{3}$.

The **point** is $\left(\frac{2}{3};\ \mathbf{0}\right)$

y = 3x + 1:

$\therefore\ y = 3(0) + 1$
$= 1$

$\therefore$ The graph cuts the **y–axis** at 1.

The **point** is (**0**; 1)

$\therefore\ 0 = 3x + 1$

$\therefore\ 3x = -1$

$\therefore\ x = -\frac{1}{3}$

$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $-\frac{1}{3}$.

The **point** is $\left(-\frac{1}{3};\ \mathbf{0}\right)$

4.2 They are parallel ◂ ... *they have equal gradients*

5.1 **AD:** The gradient $= -\frac{4}{2} = -2$ ... $\therefore\ m = -2$

& the y–intercept is 4 ... $\therefore\ c = 4$

$\therefore$ The equation is $y = -2x + 4$ ◂ ... $m = -2$ & $c = 4$ in $y = mx + c$

BC: The gradient $= -\frac{4}{2} = -2$ ... $\therefore\ m = -2$

& the y–intercept is -4 ... $\therefore\ c = -4$

$\therefore$ The equation is $y = -2x - 4$ ◂ ... $m = -2$ & $c = -4$ in $y = mx + c$

The standard form of the equation of a straight line is $\mathbf{y = mx + c}$, where **m** = the gradient and **c** = the y–intercept.

5.2 They are parallel.
They both have gradients of -2.

Both gradients are negative and are measured as $\frac{\textit{number of units down}}{\textit{number of units across}}$

i.e. $\frac{\textit{vertical change}}{\textit{horizontal change}}$

6.1 The gradient $= -\frac{5}{1} = \mathbf{-5}$ ◂

By inspection
- negative gradient
- $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{vertical change}}{\text{horizontal change}}$

So, — and $\frac{\textbf{5 units down}}{\textbf{1 unit across}}$

The use of a formula for the gradient is not ideal for grade 9 learners.

6.2 $\mathbf{y = -5x + 5}$ ◂ ... *gradient,* $m = -5$ & *y–intercept,* $c = 5$

6.3 The gradient of any other straight line drawn parallel to this line is -5. ◂ ... *parallel lines have the same gradient*

7.1

	A	B	C
x–coordinate	**0**	2	4
y–coordinate	-2	**0**	2

Note: $\boldsymbol{x = 0}$ *on the* ***y–axis*** *(at A)*
& $\boldsymbol{y = 0}$ *on the* ***x–axis*** *(at B)*

7.2 $\mathbf{y = x - 2}$ ◂ ... By inspection: The y–coordinates are all 2 less than the x–coordinates.

or: Gradient $= +\frac{2}{2} = 1$
& y–intercept, $c = -2$

8.1 The equation of CD: $\boldsymbol{x = 2}$ ◂

... *because every point on (vertical) line CD has an x–coordinate equal to 2*

$\therefore$ *$x = 2$ is the* ***equation*** *of CD*

The equation of a line is a 'rule' which is true for all points on the line.

8.2 The equation of AB: $\mathbf{y = 2x}$ ◂

Method 1:
Observe various points on the graph:
e.g. $(-2; -4)$; $(-1; -2)$; $(1; 2)$
and notice that y always equals twice x

Method 2:
m, the gradient $= +\frac{2}{1} = 2$
& c, the y–intercept, is 0

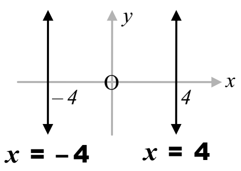
8.3 **E(2; −2)** ◂ ... *$x = 2$ and $y = -2$ at point E*

8.4 CE = **6 units** ◂ ... *CE = CD + DE = 4 + 2 = 6 units*

OR $CE = Y_C - Y_E$
$= 4 - (-2)$
$= 6$
... *the difference of the y–coordinates of C and*

9.1 The lines $x = 4$ and $x = -4$ are **parallel** to one another.

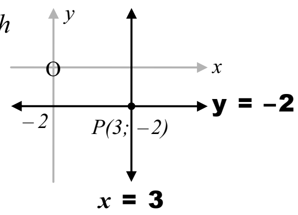
. . . The lines $\boldsymbol{x = 4}$ *and* $\boldsymbol{x = -4}$*: are both parallel to the y–axis*



9.2 The equation of the horizontal line through the point $P(3; -2)$ is $\mathbf{y = -2}$.

. . . The horizontal line through P(3; –2) is $\boldsymbol{y = -2}$*;*

The vertical line through P(3; –2) is $\boldsymbol{x = 3}$*;*

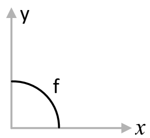


9.3 The gradient of the line defined by $y - 4x + 5 = 0$ is equal to **4**.

$$\ldots\ y - 4x + 5 = 0$$
$$\therefore\ y = \mathbf{4}x - 5 \qquad \ldots\ y = \mathbf{m}x + c$$

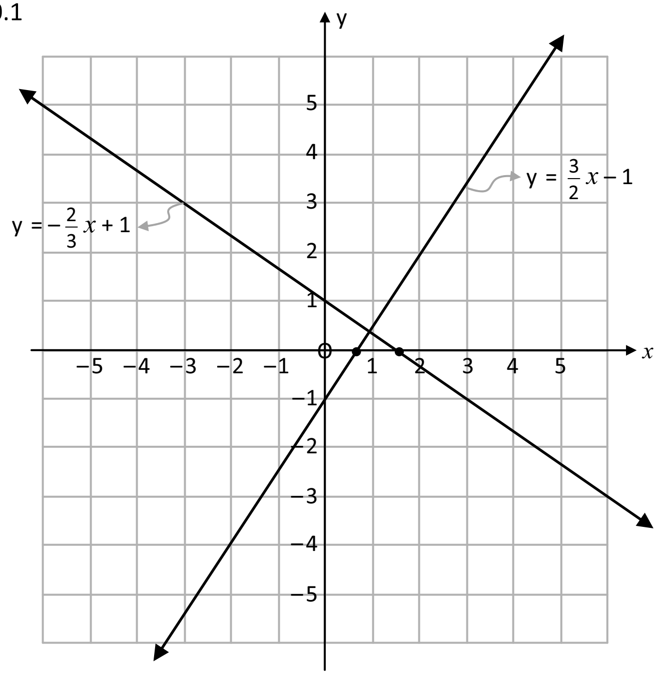
$\therefore$ *The gradient, which is the coefficient of x, is* **4**

9.4 This graph of f below represents a **non–linear** function.



. . . A linear function is a straight line, not a curve.


10.1



To find the points where the graphs cut the axes:

$$\mathbf{y = -\frac{2}{3}x + 1:} \qquad\qquad \mathbf{y = \frac{3}{2}x - 1:}$$

For the **Y-intercept**, substitute $\boldsymbol{x = 0}$

$y = -\frac{2}{3}x + 1$	$y = \frac{3}{2}x - 1$
$\therefore\ y = -\frac{2}{3}(0) + 1$	$\therefore\ y = \frac{3}{2}(0) - 1$
$= 1$	$= -1$
$\therefore$ The graph cuts the **y–axis** at 1.	$\therefore$ The graph cuts the **y–axis** at -1.
The **point** is $(\mathbf{0}; 1)$	The **point** is $(\mathbf{0}; -1)$

For the **X-intercept**, substitute **y = 0**

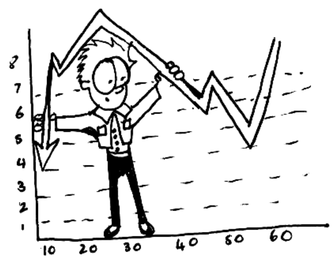
$y = -\frac{2}{3}x + 1$	$y = \frac{3}{2}x - 1$
$\therefore\ 0 = -\frac{2}{3}x + 1$	$\therefore\ 0 = \frac{3}{2}x - 1$
$\therefore\ \frac{2}{3}x = 1$	$\therefore\ \frac{3}{2}x = 1$
$\therefore\ 2x = 3 \quad \ldots \times 3$	$\therefore\ 3x = 2 \quad \ldots \times 2$
$\therefore\ x = \frac{3}{2} \quad \ldots \div 2$	$\therefore\ x = \frac{2}{3} \quad \ldots \div 3$
$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{3}{2}$.	$\therefore$ The graph cuts the $\boldsymbol{x}$**–axis** at $\frac{2}{3}$.
The **point** is $\left(\frac{3}{2}; \mathbf{0}\right)$	The **point** is $\left(\frac{2}{3}; \mathbf{0}\right)$

10.2 They are perpendicular.

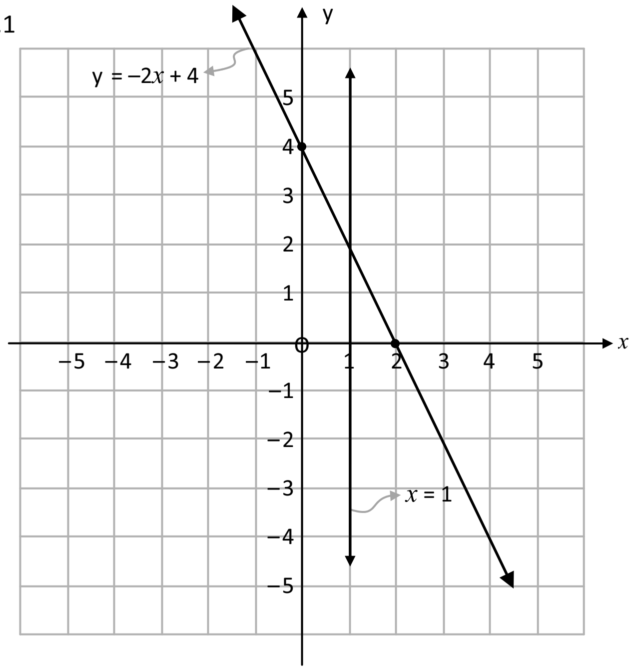
Out of interest:

Compare the gradients, $-\frac{2}{3}$ and $\frac{3}{2}$.

They are negative inverses of one another.



11.1



To find the points where the graphs cut the axes :

y = −2x + 4:

y–intercept (substitute **x = 0**) : $\quad y = -2(\mathbf{0}) + 4$
$\qquad = 4$

x–intercept (substitute **y = 0**) : $\quad \mathbf{0} = -2x + 4$
$\therefore\ 2x = 4$
$\therefore\ x = 2$

x = 1: This graph is a **vertical** line through $x = 1$. Every point on the graph has an x–coordinate equal to 1.

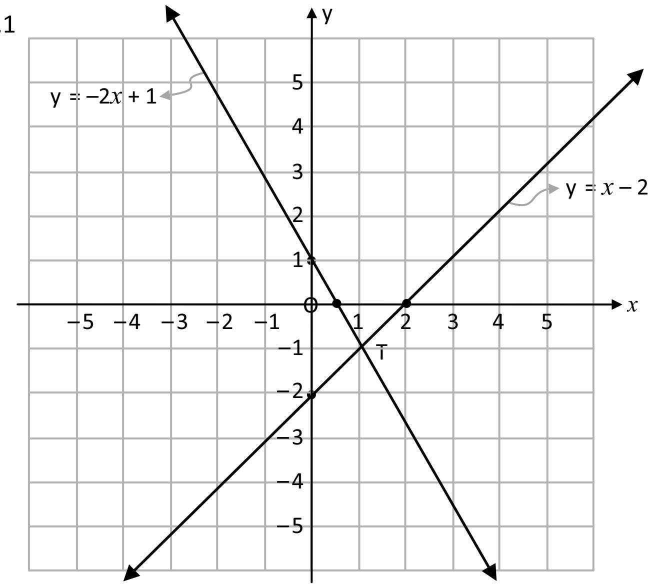
11.2 The point of intersection is (1; 2) ➤

. . . *At this point, $x = 1$ and $y = -2x + 4$*
(i.e. both equations are true)

$\therefore\ y = -2(1) + 4$
$\qquad = 2$

$\therefore$ *The point is (1; 2)*

12.1



To find the points where the graphs cut the axes :

y = −2x + 1:	**y = x − 2:**
For the **Y-intercept**, substitute **x = 0**	
$\therefore\ y = -2(0) + 1$ $= 1$	$\therefore\ y = (0) - 2$ $= -2$
$\therefore$ The graph cuts the **y–axis** at 1.	$\therefore$ The graph cuts the **y–axis** at −2.
The **point** is (**0**; 1)	The **point** is (**0**; −2)
For the **X-intercept**, substitute **y = 0**	
$\therefore\ 0 = -2x + 1$ $\therefore\ 2x = 1$ $\therefore\ x = \frac{1}{2}$	$\therefore\ 0 = x - 2$ $\therefore\ x = 2$
$\therefore$ The graph cuts the **x–axis** at $\frac{1}{2}$.	$\therefore$ The graph cuts the **x–axis** at 2.
The **point** is $\left(\frac{1}{2};\ \mathbf{0}\right)$	The **point** is (2; **0**)

12.2 $y = -2x + 1$ . . . ❶ & $y = x - 2$ . . . ❷

❶ = ❷ : $\quad -2x + 1 = x - 2$
$\therefore\ -2x - x = -2 - 1$
$\therefore\ -3x = -3$
$\therefore\ x = 1$

Both equations must be true at T, the point of intersection.

Substitute $x = 1$ into ❷ : $\quad y = (1) - 2$ [OR into ❶ !]
$\qquad = -1$

$\therefore$ T(1; −1) ➤

NOTES