Gr 9 Maths: Content Area 3 & 4
Geometry & Measurement (2D)
ANSWERS

- Geometry of Straight Lines
- Triangles: Basic facts
- Congruent Δ^s
- Similar Δ^s
- Quadrilaterals
- Polygons

Theorem of Pythagoras

Area and Perimeter of 2D shapes

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GEOMETRY OF STRAIGHT LINES

1.1 $\hat{a} = 43^o$ ◄ . . . *vertically opposite* $\angle^s$

$\hat{b} = \hat{a}$ . . . *corresponding* $\angle^s$; *AB || CD*

$= 43^o$ ◄

1.2 $\hat{c} = 180^o - (12^o + 58^o)$ . . . *adjacent* $\angle^s$ *on a straight line add up to* 180^o

$= 110^o$ ◄

1.3 $P\hat{Q}R = 112^o$ . . . *alternate* $\angle^s$; *PQ || SRT*

$\therefore \hat{d} = 180^o - 112^o$ . . . *co-interior* $\angle^s$ *supplementary; PS || QR*

$= 68^o$ ◄

---

2. $x - 9^o + x - 6^o + x + 15^o = 360^o$ . . . $\angle^s$ *about a point add up to* 360^o

$\therefore 3x = 360^o$

$\therefore x = 120^o$ ◄

$\therefore$ The largest angle $= x + 15° = 135°$ ◄

---

3. complementary 3.2 360^o ◄

4.1 $\hat{D} + \hat{F} = 90^o$ ◄ 4.2 180^o ◄

4.2 360^o ◄ 4.4 parallel ◄

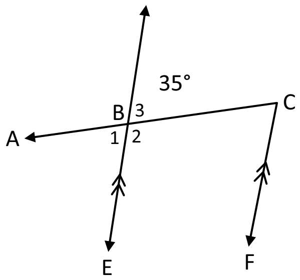
4.5 equal ◄

---

5. $\hat{B}_1\ (= \hat{B}_3) = 35^o$ ◄ . . . *vertically opposite* $\angle^s$

$B\hat{C}F = \hat{B}_1$ . . . *corresponding* $\angle^s$; *BE || CF*

$= 35^o$ ◄



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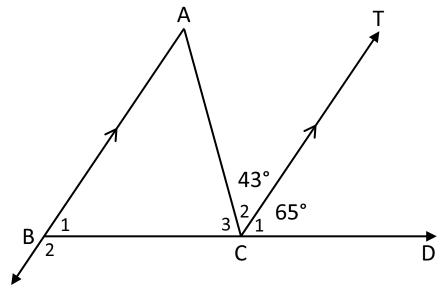


$\hat{A} = \hat{C}_2$ . . . *alternate* $\angle^s$; *AB || TC*

$= 43^o$ ◄

$\hat{B}_1 = \hat{C}_1$ . . . *corresponding* $\angle^s$; *AB || TC*

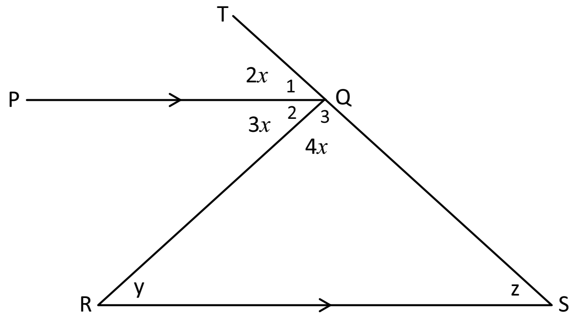
$= 65^o$ ◄

$\hat{B}_2 = 180^o - \hat{B}_1$ . . . $\angle^s$ *on a straight line*

$= 115^o$ ◄

Be sure to study
'Straight Line Geometry' (page Q2)
- vocabulary and facts -
and 'More Straight Line Geometry' (page Q13)
- vocabulary and facts -

7.



7.1 $2x + 3x + 4x = 180^o$ . . . $\angle^s$ *on a straight line*

$\therefore 9x = 180^o$

$\therefore x = 20^o$ ◄

7.2 $y\ (= \hat{Q}_2) = 3x$ . . . *alternate* $\angle^s$; *PQ || RS*

$= 60^o$ ◄ . . . $x = 20^o$ *in Question 7.1*

7.3 $z\ (= \hat{Q}_1) = 2x$ ◄ . . . *corresponding* $\angle^s$; *PQ || RS*

$= 40^o$ ◄

---

8. $(3x - 10^o) + (x + 30^o) = 180^o$ . . . *co-interior* $\angle^s$; *AB || CD*

$\therefore 4x + 20^o = 180^o$

Subtract 20^o: $\therefore 4x = 160^o$

Divide by 4: $\therefore x = 40^o$ ◄

---

9. $P\hat{U}V = 180^o - 76^o$ . . . $\angle^s$ *on a straight line*

$= 104^o$

$\therefore R\hat{V}W = P\hat{U}V$

$\therefore$ PQ || RS ◄ . . . *corresponding* $\angle^s$ *equal*

10. $\hat{a} = 60°$ ◄ . . . *vertically opposite angles*

$\hat{b} = 35°$ ◄ . . . *alternate* $\angle^s$; || *lines*

$\hat{c} = 35°$ ◄ . . . *base* $\angle^s$ *of isosceles* Δ

$\hat{d} = 180° - (\hat{a} + \hat{c})$ . . . *sum of the* $\angle^s$ *of a* Δ

$= 180° - (60° + 35°)$

$= 85°$ ◄

$\hat{e} = \hat{a} - 35°$ . . . *exterior* $\angle$ *of a* Δ = *the sum of the interior opposite* $\angle^s$

$= 25°$ ◄

$\hat{f} = (\hat{b} + \hat{c})$ . . . *corresponding* $\angle^s$; || *lines*

$= 70°$ ◄

or $\hat{f} = \hat{c} + 35°$ . . . *ext* $\angle$ *of* Δ = *sum of int. opp.* $\angle^s$

$= 70°$ ◄

$\hat{g} = \hat{e}$ . . . *alternate* $\angle^s$; || *lines*

$= 25°$ ◄

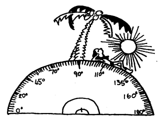
---

11. $x + (x + y) + y = 180°$ . . . $\angle^s$ *on a straight line*

$\therefore\ 2x + 2y = 180°$

i.e. $\therefore\ 2(x + y) = 180°$

Divide by 2: $\therefore\ x + y = 90°$ ◄



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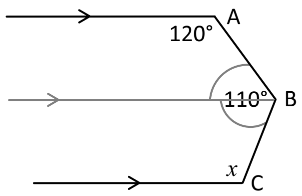
12. $120° + 110° + x = 2 \times 180°$

. . . *2 pairs of co-interior* $\angle^s$; *parallel lines*

$\therefore\ 230° + x = 360°$

Subtract 230°: $\therefore\ x = 130°$



NOTES

TRIANGLES: BASIC FACTS

1.1 $\hat{T}_1 = 60^\circ$ ◄ . . . *$\angle^s$ of an equilateral Δ all = 60^o*

$\therefore\ \hat{T}_2 = 120^\circ$ ◄ . . . *$\angle^s$ on a straight line are supplementary*

---

2. $\hat{E}_2 + \hat{W}_1 = 180^\circ - 70^\circ$ . . . *sum of the $\angle^s$ of a Δ*

$= 110^\circ$

But $\hat{E}_2 = \hat{W}_1$ . . . *AE = AW; $\angle^s$ opposite equal sides in an isosceles Δ*

$\therefore\ \hat{E}_2\ (= \hat{W}_1) = 55^\circ$

$\therefore\ x\ (= \hat{E}_2) = 55^\circ$ ◄ . . . *alternate $\angle^s$; CS || HN*

Often, in geometry riders, there are several possible methods.

3.1 $\hat{T}_1 = 25^\circ$ ◄ . . . *$\angle^s$ opposite equal sides MT and MP in an isosceles triangle*

3.2 $\hat{M}_2 = \hat{P} + \hat{T}_1$ . . . *exterior $\angle$ of Δ MPT*

$= 2(25^\circ)$

$= 50^\circ$ ◄

---

4. $4x + 5x = 180^\circ$ ◄ . . . *$\angle^s$ on a straight line*

$\therefore\ 9x = 180^\circ$

$\therefore\ x = 20^\circ$

$E\hat{F}C = \hat{E} + \hat{D}$

$\therefore\ 5x = \hat{E} + 3x$

OR: $\hat{E} = 5x - 3x$

$= 2x$

. . . *exterior $\angle$ of Δ equals the sum of the interior $\angle^s$*

$\therefore\ \hat{E} = 2x$

$= 40^\circ$ ◄

$\therefore$ Answer: A ◄

---

5. $\hat{B}\ (= \hat{C}) = x$ . . . *$\angle^s$ opposite equal sides in an isosceles Δ*

$\therefore\ \hat{A} = 180^\circ - 2x$ ◄ . . . *sum of the $\angle^s$ of Δ = 180^o*

---

6. $\hat{A} = 110^\circ - 50^\circ$ . . . *exterior $\angle$ of Δ = sum of interior opposite $\angle^s$*

$= 60^\circ$ ◄

$\therefore$ Answer: B ◄

NB:
An interior angle = the exterior $\angle$ – the other interior $\angle$

---

7. In ΔABD: $\hat{a} = A\hat{B}D$ . . . *the base $\angle^s$ of an isosceles Δ are equal*

$\therefore\ \hat{a} = \frac{1}{2}(180^\circ - 72^\circ)$ . . . *sum of the $\angle^s$ of a Δ*

$= \frac{1}{2}(108^\circ)$

$= 54^\circ$ ◄

$\hat{b} = 72^\circ + \hat{a}$ . . . *ext $\angle$ of ΔABD = the sum of the interior opposite $\angle^s$*

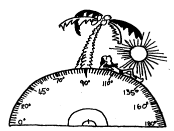
$= 126^\circ$ ◄

$\hat{c} = B\hat{D}C$ . . . *the base $\angle^s$ of an isosceles Δ are equal*

$\hat{c} = \frac{1}{2}(180^\circ - \hat{b})$

$= \frac{1}{2}(54^\circ)$

$= 27^\circ$ ◄



8.1 $x = 106^\circ - 44^\circ$ . . . *exterior $\angle$ of Δ = sum of interior opposite $\angle^s$*

$= 62^\circ$

NB: *See comment in Question 6.*

8.2 $\hat{a} + 44^\circ = 90^\circ$ . . . *sum of the $\angle^s$ of a Δ = 180^o*

$\therefore\ \hat{a} = 90^\circ - 44^\circ$

$= 46^\circ$ ◄

$\hat{b} + 28^\circ = 44^\circ$ . . . *$\angle^s$ opposite equal sides in an isosceles Δ*

$\therefore\ \hat{b} = 44^\circ - 28^\circ$

$= 16^\circ$ ◄

$\hat{c} = \hat{b} + 90^\circ$ . . . *exterior $\angle$ of Δ = sum of interior opposite $\angle^s$*

$= 16^\circ + 90^\circ$

$= 106^\circ$ ◄

---

9. $\hat{x} = 75^\circ - 44^\circ$ . . . *exterior $\angle$ of ΔBDC = sum of interior opposite $\angle^s$*

$= 31^\circ$ ◄

$\hat{y} = \hat{x}$ . . . *alternate $\angle^s$; AD || BC*

$= 31^\circ$ ◄

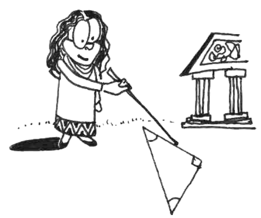
---

10. $\hat{E} = 95^\circ - 30^\circ$ . . . *exterior $\angle$ of ΔDEC = sum of interior opposite $\angle^s$*

$= 65^\circ$ ◄

$\hat{A} = 180^\circ - \hat{E}$ . . . *co-interior $\angle^s$ are supplementary because AB || ED*

$= 115^\circ$ ◄



CONGRUENT TRIANGLES
(Symbol: ≡)

1.1 ΔDEF ➤ . . . $S\angle S$

NB: *The letters must be in the correct order so that equal sides and angles correspond.*

2. ΔSTV ➤ . . . $S\angle S$

3.1 $S\angle S$ ➤ ∴ Answer : C ➤

4. In ΔABD and ΔCDB

(1) $\hat{B}_2 = \hat{D}_1 = 90°$ . . . *given*

(2) AD = CB . . . *given*

(3) BD is common

∴ ΔABD ≡ ΔCDB ➤ . . . *90°, Hyp, S (RHS)*

Note:
*Observe the **layout** of a congruency proof.*

5.1 In ΔABD and ΔACD

(1) AB = AC . . . *given*

(2) BD = CD . . . *given*

(3) AD is common

∴ ΔABD ≡ ΔACD ➤ . . . *SSS*

5.2 ∴ $\hat{A}_1 = \hat{A}_2$ . . . **corresponding $\angle^s$ of congruent Δ^s in Question 5.1*

i.e. DA bisects $B\hat{A}C$ ➤

** Nothing to do with corresponding $\angle^s$ on || lines*

6. In ΔKNQ and ΔMPQ

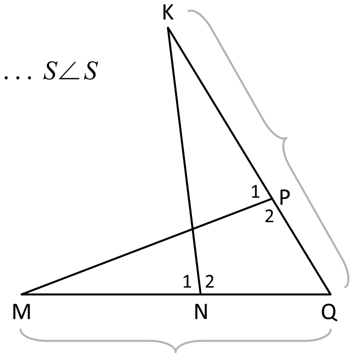
(1) NQ = PQ . . . *given*

(2) KQ = MQ . . . *given*

(3) $\hat{Q}$ is common

∴ ΔKNQ ≡ ΔMPQ ➤ . . . $S\angle S$

NB: Always give reasons!



7.1 BE = BD + **DE**

& CD = EC + **DE**

But : BD = EC . . . *given*

∴ BE = CD ➤

NB: Always give reasons!

7.2 In ΔABE and ΔACD

(1) BE = CD . . . *proved in Question 7.1*

(2) $A\hat{E}B = A\hat{D}C$ . . . *$\angle^s$ opposite equal sides in isosceles ΔADE*

(3) AE = AD . . . *given*

∴ ΔABE ≡ ΔACD . . . $S\angle S$

∴ Answer: ΔACD ➤

Study the proof carefully!

***Order** is important in congruency layout :*

- *The **letters** must be in the same order in both triangles, corresponding to the equal sides and angles of the triangles;*
- *In the facts (1), (2) and (3), the **sides and angles** of the **first** triangle must come first.*

8.1 *We need to prove congruent triangles!*

In ΔMPN and ΔMTN

(1) MP = MT . . . *radii of the circle*

(2) MN is common

(3) $\hat{N}_2 = \hat{N}_1 = 90°$ . . . *given that MN ⊥ PT*

∴ ΔM**PN** ≡ ΔM**TN** ➤ . . . *RHS*

∴ PN = NT ➤ . . . *corresponding sides of congruent triangles*

9.1 BC = BF + **FC**

& EF = CE + **FC**

But : BF = CE . . . *given*

∴ BC = EF ➤

9.2 In ΔABC and ΔDEF

(1) AB = DE . . . *given*

(2) AC = DF . . . *given*

(3) BC = EF . . . *proved in Question 9.1*

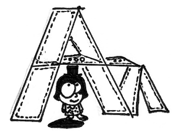
∴ ΔABC ≡ ΔDEF ➤ . . . *SSS*

9.3 $\hat{B} = \hat{E}$ because they are corresponding angles of the congruent triangles in Question 9.2 ➤

9.4 $\hat{B}$ and $\hat{E}$ are alternate angles

& $\hat{B} = \hat{E}$ in Question 9.3

∴ AB || ED . . . *converse fact*



See the notes on Congruency and Similarity on page A5

10.1 In ΔABD and ΔACD

(1) AB = AC . . . *given*

(2) BD = CD . . . *given*

(3) AD is common

$\therefore$ $\Delta ABD \equiv \Delta ACD$ ◄ . . . *SSS*

10.2 In ΔABE and ΔACE

(1) AB = AC . . . *given*

(2) $\hat{A}_1 = \hat{A}_2$ . . . *corresponding angles in congruent triangles in Question 10.1*

(3) AE is common

$\therefore$ $\Delta ABE \equiv \Delta ACE$ ◄ . . . $S\angle S$

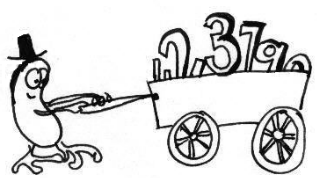
10.3 $\hat{E}_1 = \hat{E}_2$ . . . *corresponding* $\angle^s$ *of congruent triangles in Question 10.2*

But $\hat{E}_1 + \hat{E}_2 = 180^\circ$ . . . *angles on a straight line*

$\therefore$ $\hat{E}_1 = \hat{E}_2 = 90^\circ$ ◄

Study the (easy) ***logic*** *very carefully!*

10.4 AE $\perp$ BC ◄ [i.e. AE is perpendicular to BC ◄]

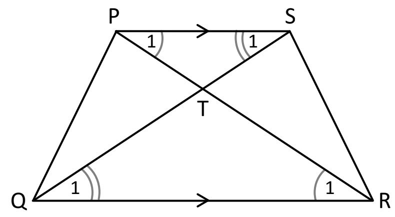


$\equiv$ *means 'is* ***congruent*** *to'*
(i.e. same ***SHAPE*** *and* ***SIZE****)*

whereas:

||| *means 'is* ***similar*** *to'*
(i.e. same ***SHAPE*** *but not necessarily same* ***SIZE***

11. The sketch with all equal angles filled in.
(There are no equal sides)



Answer: D ◄

Proof: In ΔPTS and ΔRTQ

(1) $\hat{P}_1 = \hat{R}_1$ . . . *alternate* $\angle^s$*;* $PS \parallel QR$

(2) $\hat{S}_1 = \hat{Q}_1$ . . . *alternate* $\angle^s$*;* $PS \parallel QR$

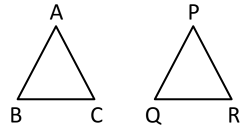
[& (3) $P\hat{T}S = R\hat{T}Q$ . . . *vertically opposite* $\angle^s$]

$\therefore$ $\Delta PTS \ ||| \ \Delta RTQ$ ◄ . . . $\angle\angle\angle$

Congruency ($\equiv$) and Similarity (|||) of triangles

Congruent Triangles . . .

have the same **shape** and **size**.
All 3 angles and all 3 sides are equal.



i.e. $\Delta ABC \equiv \Delta PQR$ means that
$\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$

and AB = PQ, AC = PR and BC = QR

Note the order of the lettering

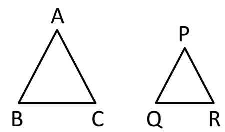
Two triangles are **congruent** if they have

- 3 sides the same length . . . **SSS**
- 2 sides & an included angle equal . . . **S$\angle$S**
- a right angle, hypotenuse & **a** side equal . . . **RHS**
- 2 angles and a side equal . . . $\angle\angle$**S**

Similar Triangles . . .

have the same **shape**, but not necessarily the same size.

All 3 angles are equal.



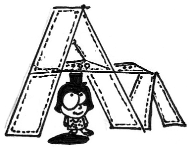
i.e. $\Delta ABC \ ||| \ \Delta PQR$ means that
$\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$

The sides are not necessarily equal, but are proportional:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

Note the order of the lettering

SIMILAR $\Delta^{'s}$ (Symbol: |||)



Be sure to read the notes on Congruency and Similarity on pg. A5

1.1 If $\Delta DEF \parallel\!\mid \Delta KLM$, i.e. ΔDEF is **similar** to ΔKLM,

then $\dfrac{DE}{KL} = \dfrac{EF}{LM} = \dfrac{DF}{\mathbf{KM}}$ ◄ ... *proportional sides of similar triangles*

$\therefore \dfrac{14}{7} = \dfrac{x}{\mathbf{12}}$ ... $\dfrac{14}{7}\left(=\dfrac{2}{1}\right) = \dfrac{\mathbf{?}}{12}$

$\therefore x = \mathbf{24}$ ◄

Note the ***ORDER of the letters*** *in similar Δ^s:*
ΔDEF ||| ***ΔKLM*** → $\hat{D} = \hat{K}$, $\hat{E} = \hat{L}$ and $\hat{F} = \hat{M}$
and this determines the proportional sides

---

2. If ΔABC ||| ΔEDF,

then $\dfrac{AB}{ED} = \dfrac{BC}{DF} \left(= \dfrac{AC}{EF}\right)$ ... *proportional sides of similar triangles*

$\therefore \dfrac{AB}{6} = \dfrac{15}{10}$

Choose the sides for which you have the lengths!

Multiply by 6 :

$\therefore AB = \dfrac{15 \times 6}{10}$

$\therefore AB = 9$ cm ◄

3.1 In ΔPQR and ΔSTR

(1) $\hat{P} = \hat{S}$ ... *alternate $\angle^s$; PQ || ST*

(2) $\hat{Q} = \hat{T}$ ... *alternate $\angle^s$; PQ || ST*

[& (3) $P\hat{R}Q = S\hat{R}T$ ... *vertically opposite $\angle^s$*]

$\therefore \Delta PQR$ ||| ΔSTR ◄ ... $\angle\angle\angle$

3.2 $\therefore \dfrac{PQ}{ST} = \dfrac{PR}{SR} \left(= \dfrac{QR}{TR}\right)$ ... *proportional sides of similar triangles*

$\therefore \dfrac{PQ}{3} = \dfrac{10}{6}$

Choose the sides for which the lengths have been given.

Multiply by 3 :

$\therefore PQ = \dfrac{10 \times 3}{6}$

$\therefore PQ = 5$ cm ◄

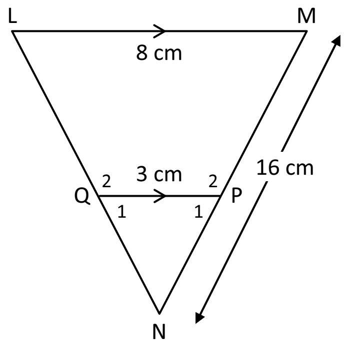
4.1 In ΔQPN and ΔLMN

(1) $\hat{N}$ **is common**

(2) $\hat{P}_1 = \hat{\mathbf{M}}$ ... ***corresponding $\angle^s$; QP || LM***

[& (3) $\hat{Q}_1 = \hat{\mathbf{L}}$ ... ***corresponding $\angle^s$; QP || LM***]

$\therefore \Delta QPN$ ||| $\mathbf{\Delta LMN}$ ... $\angle\angle\angle$



4.2 $\therefore \dfrac{PN}{MN} = \dfrac{QP}{LM} \left(= \dfrac{QN}{LN}\right)$ ... *proportional sides of similar triangles*

$\therefore \dfrac{PN}{16} = \dfrac{3}{8}$

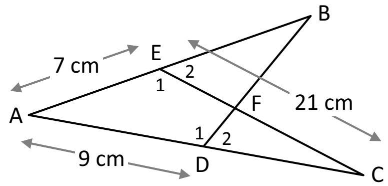
Choose the sides for which the lengths have been given.

Multiply by 16 :

$\therefore PN = \dfrac{3 \times \cancel{16}^{\,2}}{\cancel{8}}$

$\therefore PN = 6$ cm ◄

5.1 In ΔABD and ΔACE



(1) $\hat{A}$ is common

(2) $A\hat{\mathbf{B}}D = A\hat{\mathbf{C}}E$

[(3) 3rd L : $\hat{D}_1 = \hat{E}_1$ ... *sum of the $\angle^s$ of a Δ*]

$\therefore \Delta ABD$ ||| ΔACE ◄ ... $\angle\angle\angle$

5.2 $\therefore \dfrac{BD}{CE} = \dfrac{AD}{AE} \left(= \dfrac{AB}{AC}\right)$ ... *proportional sides of similar triangles*

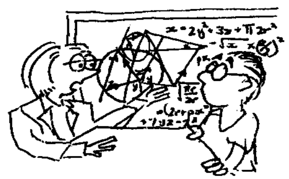
$\therefore \dfrac{BD}{21} = \dfrac{9}{7}$

Choose the sides with known lengths.

Multiply by 21 :

$\therefore BD = \dfrac{9 \times \cancel{21}^{\,3}}{\cancel{7}}$

$\therefore BD = 27$ cm ◄



QUADRILATERALS

1. $(x + 50^\circ) + (2x - 20^\circ) = 180^\circ$ . . . *co-interior* $\angle^s$; *AB || CD*

$\therefore\ 3x + 30^\circ = 180^\circ$

Subtract 30°: $\therefore\ 3x = 150^\circ$

Divide by 3: $\therefore\ x = 50^\circ$

$\therefore\ \hat{A} = 50^\circ + 50^\circ = 100^\circ$

$\therefore\ \hat{B} = 180^\circ - 100^\circ$ . . . *co-interior* $\angle^s$; *AC || BD*

$= 80^\circ$ ➤

OR: $\hat{C} = 2(50^\circ) - 20^\circ = 80^\circ$

$\therefore\ \hat{B} = 80^\circ$ ➤ . . . *opposite* $\angle^s$ *of a* $||^m$ *are equal*

2.1 $E\hat{F}G = 180^\circ - 156^\circ$ . . . *co-interior* $\angle^s$; *DE || GF in rhombus*

$= 24^\circ$ ➤

2.2 $\hat{F}_2 = \frac{1}{2}(24^\circ)$ . . . *the diagonals of a rhombus bisect the* $\angle^s$ *of the rhombus*

$= 12^\circ$ ➤

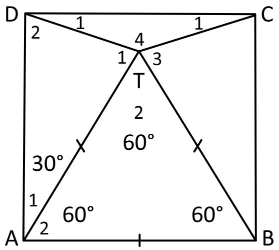
2.3 $\hat{G} = 156^\circ$ ➤ . . . *opposite* $\angle^s$ *of a rhombus (or* $||^m$*) are equal*

3.1 ΔATD and ΔBTC ➤

. . . *AT = AB* . . . *sides of equilateral* Δ

= AD . . . *sides of square*

& *Similarly : BT = AB = BC*



3.2 $\hat{A}_2 = 60^\circ$ . . . $\angle$ *of equilateral* Δ

$\therefore\ \hat{A}_1 = 30^\circ$ . . . $D\hat{A}B = 90^\circ$ *in square*

$\therefore\ \hat{D}_2 + \hat{T}_1 = 180^\circ - 30^\circ$ . . . *sum of the* $\angle^s$ *of a* Δ

$= 150^\circ$

But $\hat{D}_2 = \hat{T}_1$ . . . $\angle^s$ *opposite equal sides in* Δ *ATD*

$\therefore\ \hat{D}_2 = 75^\circ$ ➤ . . . *half of* 150°

3.3 Similarly: $\hat{T}_3 = 75^\circ$

and $\hat{T}_2 = 60^\circ$ . . . $\angle$ *of equilateral* Δ *ATB*

$\therefore\ \hat{T}_4 = 360^\circ - (75^\circ + 60^\circ + 75^\circ)$ . . . *sum of* $\angle^s$ *about a point* $= 360^\circ$

$= 360^\circ - 210^\circ$

$= 150^\circ$ ➤

OR: $\hat{D}_1 = 90^\circ - 75^\circ$ . . . $\hat{D}_2 = 75^\circ$ *and* $A\hat{D}C = 90^\circ$

$= 15^\circ$

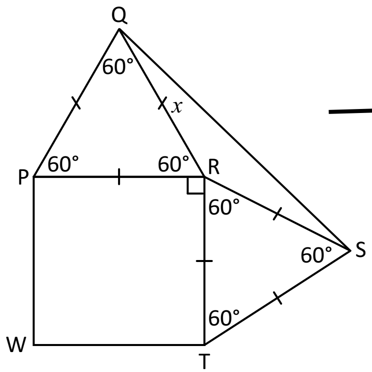
& Similarly: $\hat{C}_1 = 15^\circ$

$\therefore\ \hat{T}_4 = 180^\circ - 2(15^\circ)$ . . . *sum of the* $\angle^s$ *of* Δ *DTC*

$= 180^\circ - 30^\circ$

$= 150^\circ$ ➤

4.



$Q\hat{R}P = T\hat{R}S = 60^\circ$ . . . $\angle^s$ *of equilateral* Δ^s

& $P\hat{R}T = 90^\circ$ . . . $\angle$ *of square*

$\therefore\ Q\hat{R}S = 360^\circ - (60^\circ + 90^\circ + 60^\circ)$ . . . *sum of* $\angle^s$ *about a point* $= 360^\circ$

$= 150^\circ$

QR = PR . . . *sides of equilateral* Δ*PQR*

= RT . . . *sides of square*

= RS . . . *sides of equilateral* Δ*RTS*

$\therefore\ x = R\hat{S}Q$ . . . *angles opposite equal sides in* Δ*QRS*

$= \frac{1}{2}(180^\circ - 150^\circ)$ . . . $\angle$ *of isosceles* Δ*; sum of the* $\angle^s$ *of a* $\Delta = 180^\circ$

$= \frac{1}{2}(30^\circ)$

$= 15^\circ$ ➤

---

5. In ΔADB and ΔCBD:

$\hat{D}_1 = \hat{B}_2$ . . . *alternate* $\angle^s$; *AD || BC in parallelogram*

$\hat{B}_1 = \hat{D}_2$ . . . *alternate* $\angle^s$; *AB || DC in parallelogram*

BD = BD . . . *common side*

$\therefore\ \Delta ADB \equiv \Delta CBD$ . . . $\angle\angle S$

$\therefore$ AD = BC ➤ . . . *corresponding sides of congruent* Δ^s

& AB = DC ➤

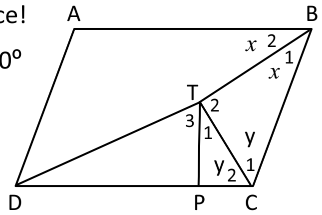
Note: *We have just proved, using congruency, that both pairs of opposite sides of a parallelogram are equal in length.*

---

6. D rectangle ➤

If one angle equals 90°*, then, because of co-interior angles and parallel lines, so do the others equal* 90°*.*
$\therefore$ *We only need* ***'at least one angle equal to 90°.'***

7.1 Something new to experience!



$(\hat{B}_1 + \hat{B}_2) + (\hat{C}_1 + \hat{C}_2) = 180°$

. . . *co-interior $\angle^s$; AB || DC in parallelogram*

Let $\hat{B}_1 = \hat{B}_2 = x$ and $\hat{C}_1 = \hat{C}_2 = y$ . . .

. . . *given that the angles were bisected*

A very useful technique in geometry!

$\therefore\ 2x + 2y = 180°$

i.e. $2(x + y) = 180°$

Divide by 2 : $\therefore\ x + y = 90°$

$\therefore$ In ΔBTC : $\hat{B}_1 + \hat{C}_1 = 90°$

$\therefore\ \hat{T}_2 = 90°$ . . . *sum of the $\angle^s$ of a Δ*

7.2 ΔTCP ➤

$T\hat{P}C = 180° - 90° = 90°$ . . . *angles on a straight line*

In ΔBCT and ΔTCB :

1) $\hat{C}_1 = \hat{C}_2$ (= y)

2) $\hat{T}_2 = T\hat{P}C$ (= 90°)

3) $\therefore\ \hat{B}_1 = \hat{T}_1$ . . . *3^{rd} $\angle$ of Δ*

$\therefore\ \Delta BCT\ |||\ \Delta TCP$ ➤ . . . $\angle\angle\angle$

7.3 $\therefore\ \dfrac{BT}{TP} = \dfrac{BC}{TC} \left(= \dfrac{CT}{CP}\right)$ . . . *proportional sides of similar triangles*

$\therefore\ \dfrac{BT}{4} = \dfrac{2TC}{TC}$

Choose the sides whose lengths you have been given.

Multiply by 4 :

$\therefore$ BT = 2×4

= 8 cm ➤

8. Answer : C $\Delta AEB \equiv \Delta DEC$ ➤

In ΔAEB and ΔDEC :

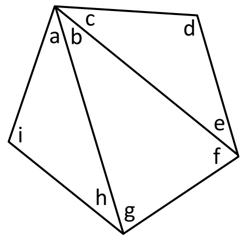
1) AE = BE . . . *given*

2) EB = EC . . . *given*

3) $A\hat{E}B = D\hat{E}C$ . . . *vertically opposite $\angle^s$*

$\therefore\ \Delta AEB \equiv \Delta DEC$ . . *S∠S*

9.



(a + i + h) + (b + g + f) + (c + d + e)

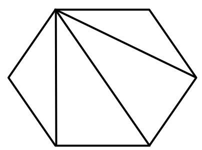
= $3 \times 180°$

= 540°

$\therefore$ Each $\angle = \dfrac{540°}{5} = 108°$

$\therefore$ Answer : D ➤

10.



The sum of the $\angle^s$ of the hexagon

= $4 \times 180°$

= 720°

$\therefore$ Each $\angle = \dfrac{720°}{6} = 120°$

$\therefore$ Answer : B ➤

NOTES

THEOREM OF PYTHAGORAS

1. AC = 13 cm . . . *5 : 12 : $\underline{13}$ Pythagoras 'trip'* *

OR : In ΔABC: $AC^2 = AB^2 + BC^2$ . . . *Thm of Pythag.; $\hat{B} = 90^o$*

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore$ AC = 13 cm

---

* ***Note:*** *When applying the Theorem of Pythagoras, there are some well-known 'trips' which are useful to know and use instead of long calculations.*

e.g. $\mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25 = \mathbf{5}^2$

$\mathbf{5}^2 + \mathbf{12}^2 = 25 + 144 = 169 = \mathbf{13}^2$

$\mathbf{8}^2 + \mathbf{15}^2 = 64 + 225 = 289 = \mathbf{17}^2$

So: *the* ***TRIP****(LET)****S****:*

3 : 4 : 5 ; ***5 : 12 : 13*** ; ***8 : 15 : 17***

and even multiples:

6 : 8 : 10

2.1 In ΔABC : x = 10 cm ◄ . . . *Pythag 'trip':* ***3 : 4 : 5 = 6 : 8 : $\underline{10}$***

2.2 In ΔABD : BD = 15 cm . . . *Pythag 'trip':* ***8 : $\underline{15}$: 17***

$\therefore$ y = 15 cm − 6 cm

= 9 cm ◄

OR: 2.1 $x^2 = 8^2 + 6^2$, etc. . . . *Theorem of Pythag*

2.2 $BD^2 = 17^2 - 8^2$, etc.

3.1 $\dfrac{TU \times 12}{2} = 30$ . . . $\dfrac{h \times b}{2}$ = *area of a Δ*

Multiply by 2 :

$\therefore$ TU $\times$ 12 = 60

Divide by 12 :

$\therefore$ TU = 5 cm ◄

. . . OR: $\dfrac{TU \times \cancel{12}^{\,6}}{\cancel{2}} = 30$

$\therefore$ 6 $\times$ TU = 30

Divide by 6:

$\therefore$ TU = 5 cm ◄

3.2 TW = 13 cm . . . *Pythag 'trip'* ***5 : 12 : $\underline{13}$***

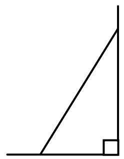
$\therefore$ The perimeter of ΔTUW = 5 cm + 12 cm + 13 cm

= 30 cm ◄

4.1 The length of the ladder = 13 m ◄ . . . *Pythag 'trip':* ***5 : 12 : $\underline{13}$***

OR : (length of the ladder)^2

= $5^2 + 12^2$, etc.



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5. Answer : B 6 cm ◄

In ΔADC : DC = 8 cm . . . *opposite sides of rectangle*

$\therefore$ AD = 6 cm . . . *Pythag 'trip':* ***3 : 4 : 5 = $\underline{6}$: 8 : 10***

---

6. This sum requires us to apply the **converse** of the Theorem of Pythagoras,

i.e. If the square on one side of a triangle equals the sum of the squares on the other two sides, then the angle opposite the first side is a right angle.

$AC^2 = 15^2 = 225$

& $AB^2 + BC^2 = 9^2 + 12^2 = 81 + 144 = 225$

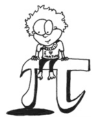
$\therefore AC^2 = AB^2 + BC^2$

$\therefore \hat{B} = 90°$ . . . *the converse of the Theorem of Pythagoras*

NOTES

MEASUREMENT: 2D

1.1 Radius, $r = \frac{1}{2} \times \text{diameter} = 6$ cm



$\therefore$ Area $= \boldsymbol{\pi r^2} = 3{,}14 \times 6^2$
$= 113{,}04$ cm^2 ◄

1.2 The circumference of the (full) circle $= \mathbf{2\pi r}$

$\therefore$ The 'circumference' of the semi-circle $= \pi r = 3{,}14 \times 6$
$= 18{,}84$ cm

$\therefore$ The perimeter of the shape ACB
$= 18{,}84$ cm + 12 cm $= 30{,}84$ cm ◄
. . . diameter, AB = 12 cm

---

2. The area of a square $= \mathbf{s^2} = (0{,}12)^2 = 0{,}0144$ cm^2

$\therefore$ Answer : D ◄

Observe carefully:
$$(0{,}12)^2 = \left(\frac{12}{100}\right)^2 = \frac{12}{100} \times \frac{12}{100} = \frac{144}{10\,000} = 0{,}0144$$

3.1 The perimeter of the field
$= 2 \times 100$ m $+$ $2 \times$ semi-circles
$= 200$ m $+$ $2 \times 3{,}14 \times 30$ m . . . *Circumference of circle* $= \boldsymbol{2\pi r}$
$= 388{,}4$ m

$\therefore$ Number of laps $= \frac{4\,000\text{ m}}{388{,}4\text{ m}} = 10{,}298$ . . .
$\therefore$ 11 laps ◄ *. . . for **at least** 4 km!*

3.2 The area of the field
$=$ area of rectangle $+$ area of 2 semi-circles
$= (100 \times 60)$m^2 $+$ $\pi.30^2$ m^2 . . . *Area of circle* $= \pi r^2$
$= 6\,000$ m^2 $+$ $\pi.900$ m^2
$\approx 8\,827{,}43$ m^2 ◄

4.1 In ΔAPS: $PS^2 = 5^2 - 2^2$ . . . *Theorem of Pythagoras*
$= 25 - 4$
$= 21$
$\therefore PS = \sqrt{21}$
$\approx 4{,}58$ m ◄

4.2 PT $= 3 \times$ AB $= 3 \times 4$ m $= 12$ m ◄

4.3 A kite ◄ *. . . 2 pairs of adjacent sides equal*

4.4 **Method 1:** Using the formula

The area $= \frac{1}{2}$ **the product of the diagonals**
$= \frac{1}{2}$ (PT $\times$ AB)
$= \frac{1}{2}$ (12×4)
$= 24$ m^2 ◄

Method 2: Without the formula

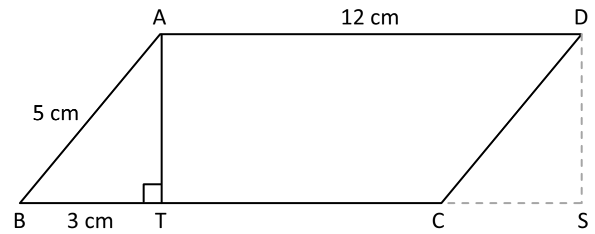
The area $= \Delta$PAT $+$ ΔPBT
$= 2 \times \Delta$PAT . . . *because the 2 Δ^s are congruent*
$= \cancel{2} \times \left(\frac{1}{\cancel{2}} \text{PT . AS}\right)$
$= 12$ m $\times$ 2 m
$= 24$ m^2 ◄

5.1 In ΔABT : AT $= 4$ cm ◄ . . . *Pythag 'trip'*: ***3 : 4 : 5***

OR: $AT^2 = 5^2 - 3^2$ . . . *Theorem of Pythagoras*
$= 25 - 9$
$= 16$
$\therefore$ AT $= 4$ cm ◄

5.2 The area of parallelogram ABCT
$=$ **base $\times$ height**
$=$ BC $\times$ AT
$=$ AD $\times$ 4 cm
$= 12$ cm $\times$ 4 cm
$= 48$ cm^2

See the shifting of ΔABT as shown :



Parallelogram ABCD = rectangle ATSD
& Area of rectangle ATSD
$=$ **length $\times$ breadth**
$= 12$ cm $\times$ 4 cm
$= 48$ cm^2

5.3.1 DC $=$ AB $= 5$ cm . . . *opposite $\angle^s$ of $||^m$*
TC $=$ BC $-$ BT $= 12$ cm $-$ 3 cm $= 9$ cm
$\therefore$ The perimeter of trapezium ADCT
$=$ AD $+$ DC $+$ TC $+$ AT
$= 12$ cm $+$ 5 cm $+$ 9 cm $+$ 4 cm
$= 30$ cm ◄

5.3.2 **Method 1:** Using the formula

The area of trapezium ADCT

$= \mathbf{\frac{1}{2}}$ **(sum of the || sides) $\times$ the distance between them**

$= \frac{1}{2}(AD + TC) \times AT$

$= \frac{1}{2}(12 + 9) \times 4$

$= 42\ cm^2$ ➤

Method 2: Without the formula

The area of trapezium ADCT

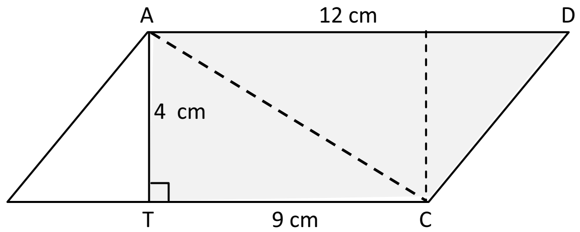
$=$ Area of ΔATC + Area of ΔADC

$= \frac{1}{2}(9 \times 4) + \frac{1}{2}(12 \times 4)$ *

$= 18 + 24$

$= 42\ cm^2$ ➤

* $\frac{1}{2}.(9 + 12).4$, like the formula above!

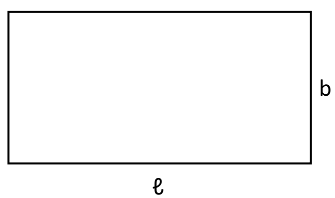


6. Let the area of the original rectangle $= \ell \times b$

If the length is doubled, then the area of the enlarged rectangle $= 2\ell \times b$

$= 2(\ell b)$

$\therefore\ k = 2$ ➤



7. The circumference of a circle, $2\pi r = 52$ cm

$\therefore\ r = \frac{52}{2\pi} = \frac{26}{\pi}$

$\therefore$ The area of the circle

$= \pi r^2 = \pi \times \left(\frac{26}{\pi}\right)^2$

$= \pi \times \frac{26^2}{\pi^2}$

$= \frac{26^2}{\pi}$

$\approx 215{,}18\ cm^2$ ➤ ... *correct to 2 decimal places*

8.1 The circumference of a circle $= 2\pi r$

$\therefore$ The circumference of the smaller circle

$= 2 \times \pi \times 20$

$\approx 125{,}66$ cm ➤

8.2 The area of the shaded region

$=$ the area of the full circle $-$ the area of the inner circle

$= \pi 30^2 - \pi 20^2$

$\approx 1\ 570{,}80\ cm^2$ ➤

9.1 The area of the shaded ring

$=$ the area of the full circle $-$ the area of the inner ring

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$ ➤

9.2 Area of the shaded ring $= \pi(14^2 - 8^2)$

$= 132\pi\ cm^2$ ➤

Note: Give the answer 'in terms of π'

NOTES